Lifting based convex approaches to labeling problems

Laurent Condat,

GIPSA-lab, Grenoble, France

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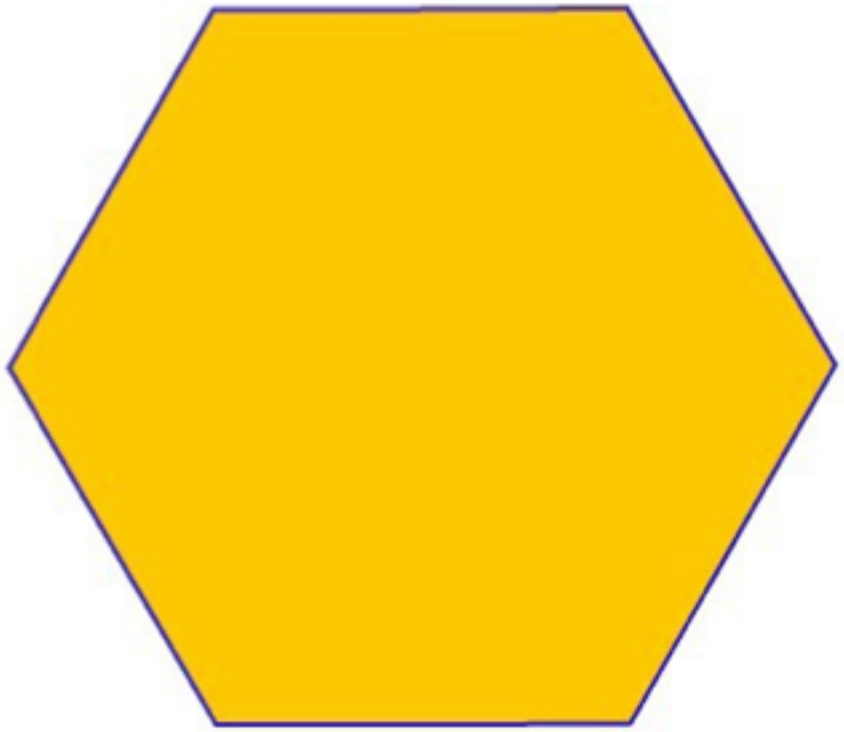
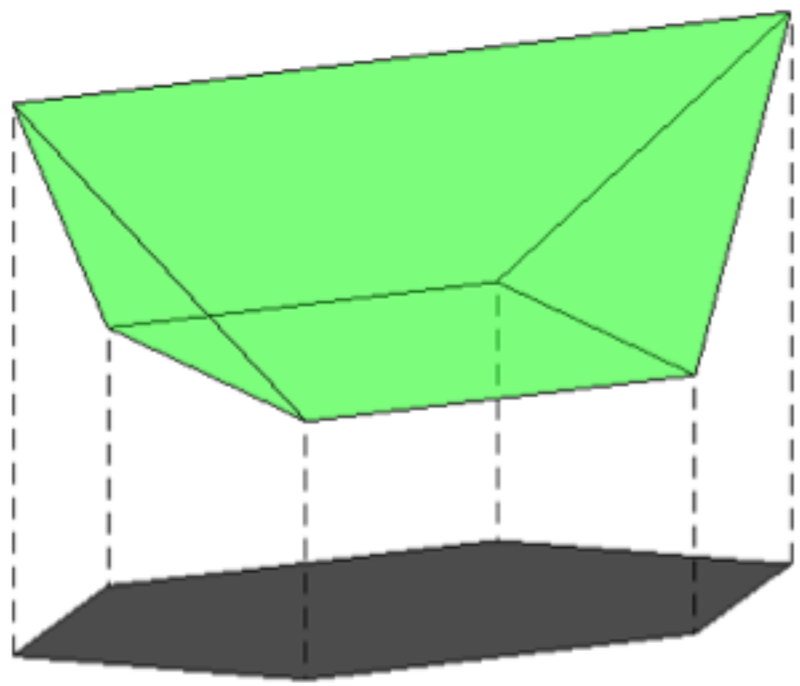
Lifting

Puzzle: describe this hexagon
with 5 linear inequalities



Thinking outside the box



[P. Parrilo]

Solution: the hexagon is the shadow of a 3-D polyhedron with 5 faces

Convex relaxations by lifting

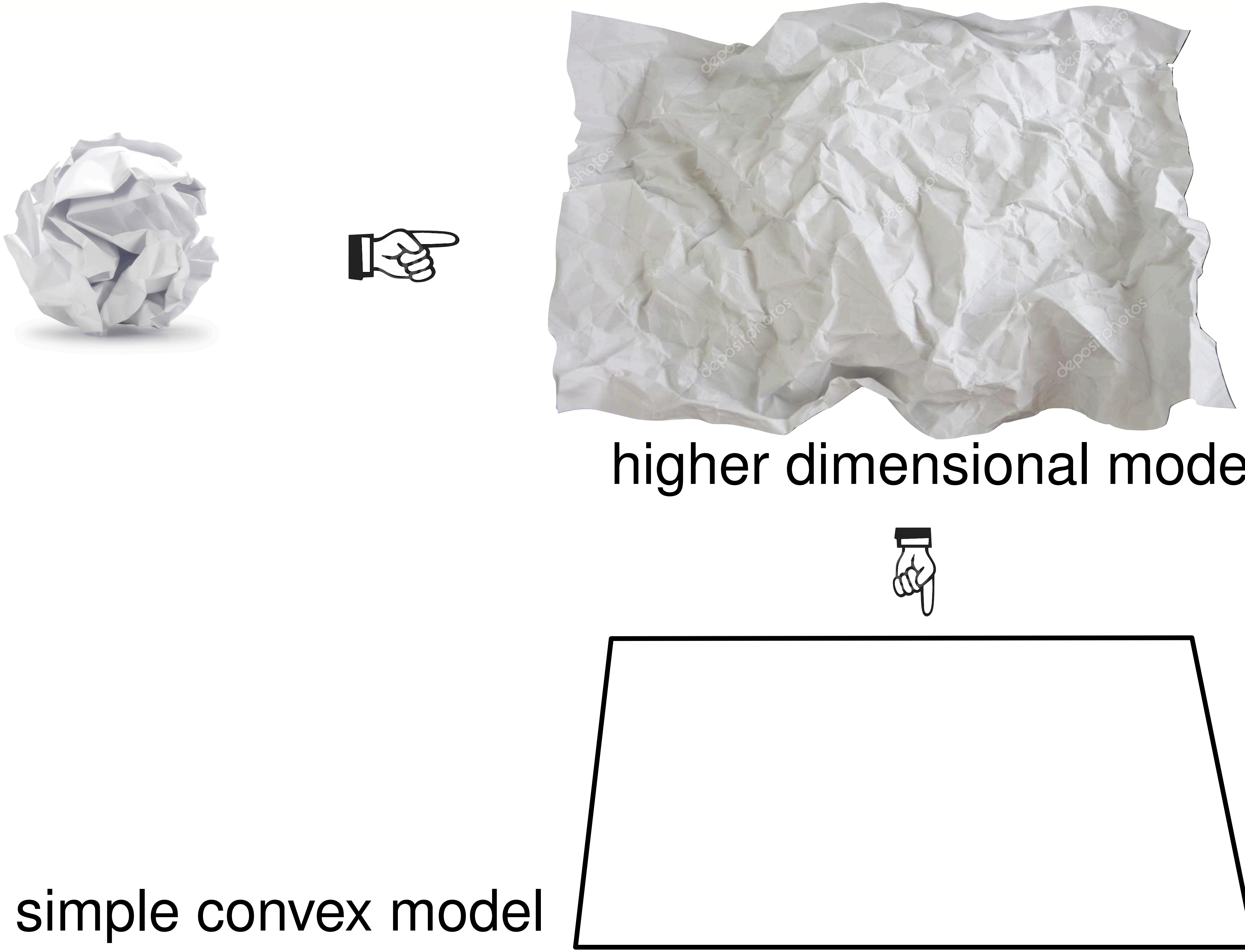
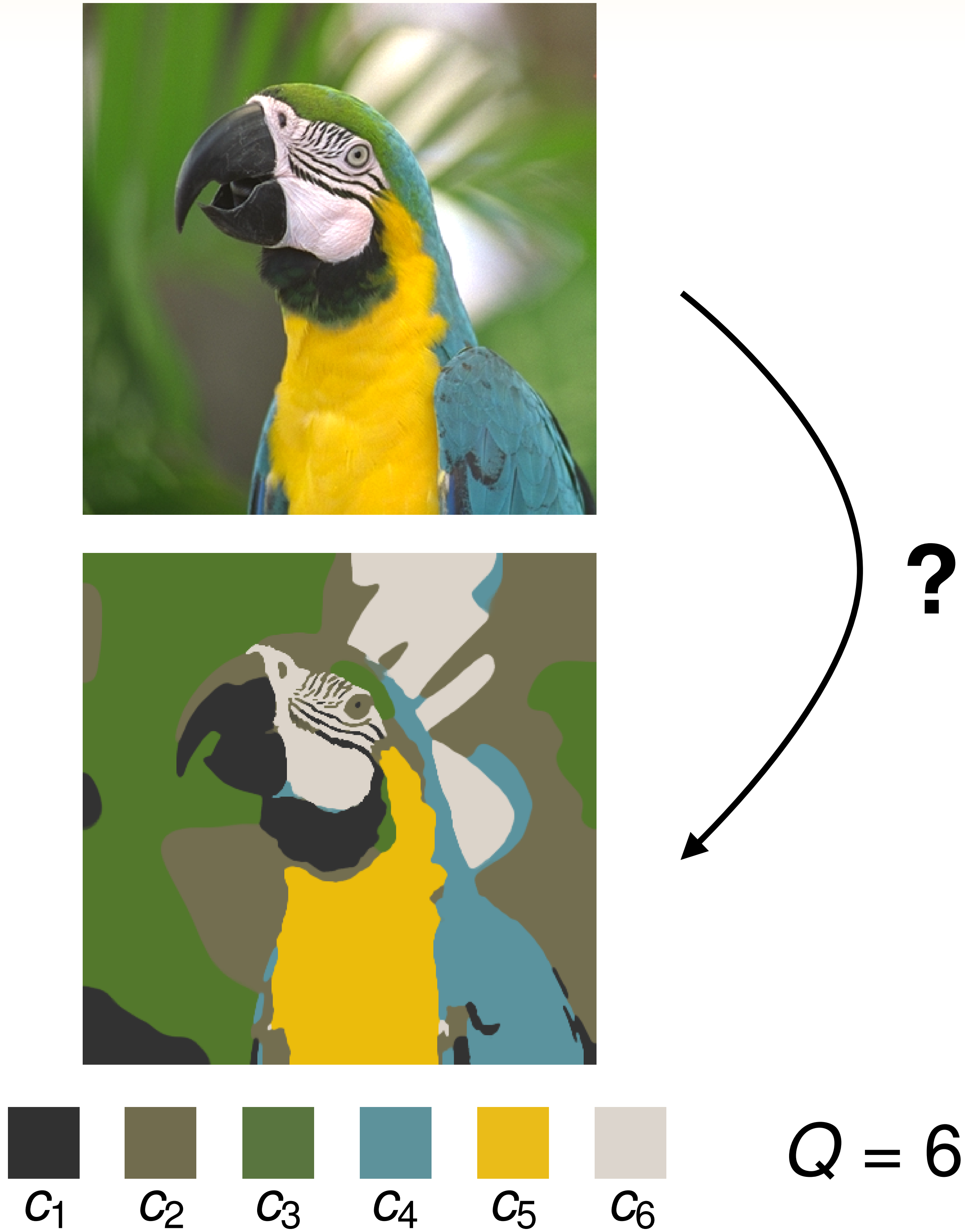
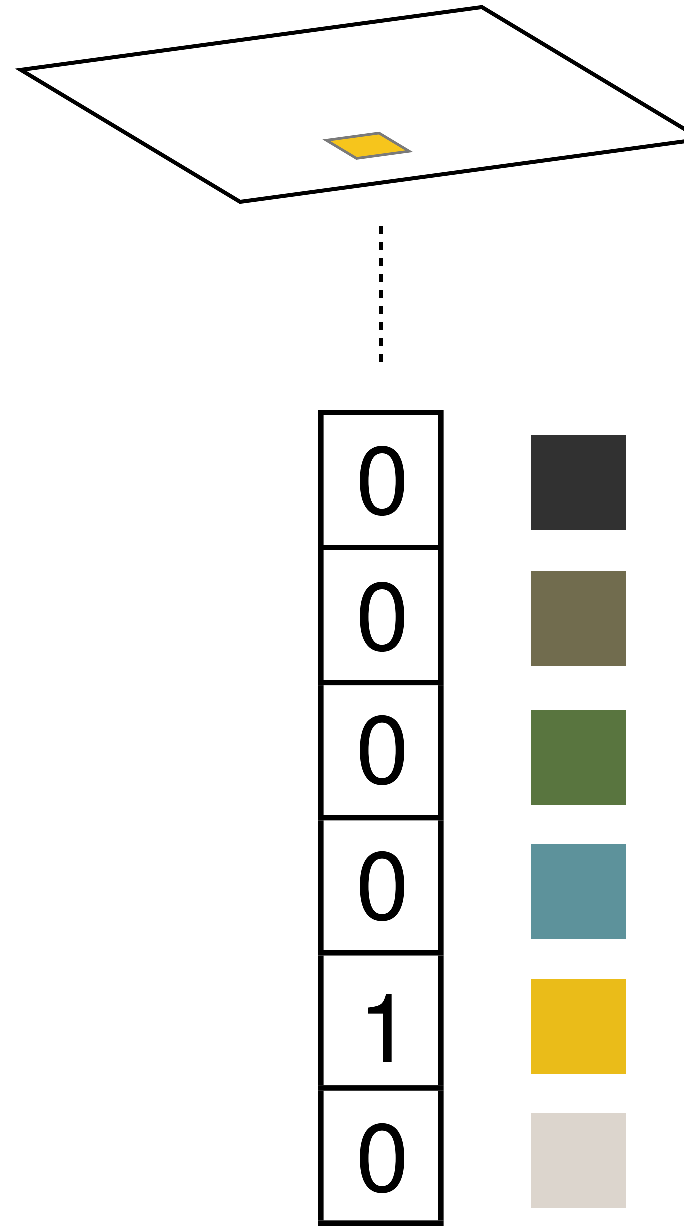


Image segmentation / Potts problem



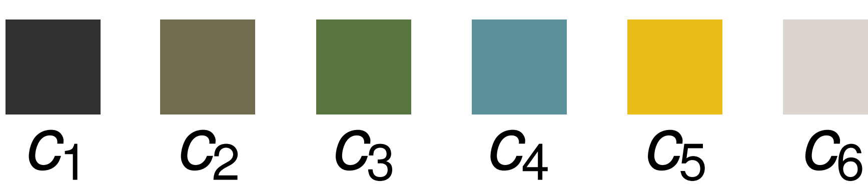
$Q = 6$ labels

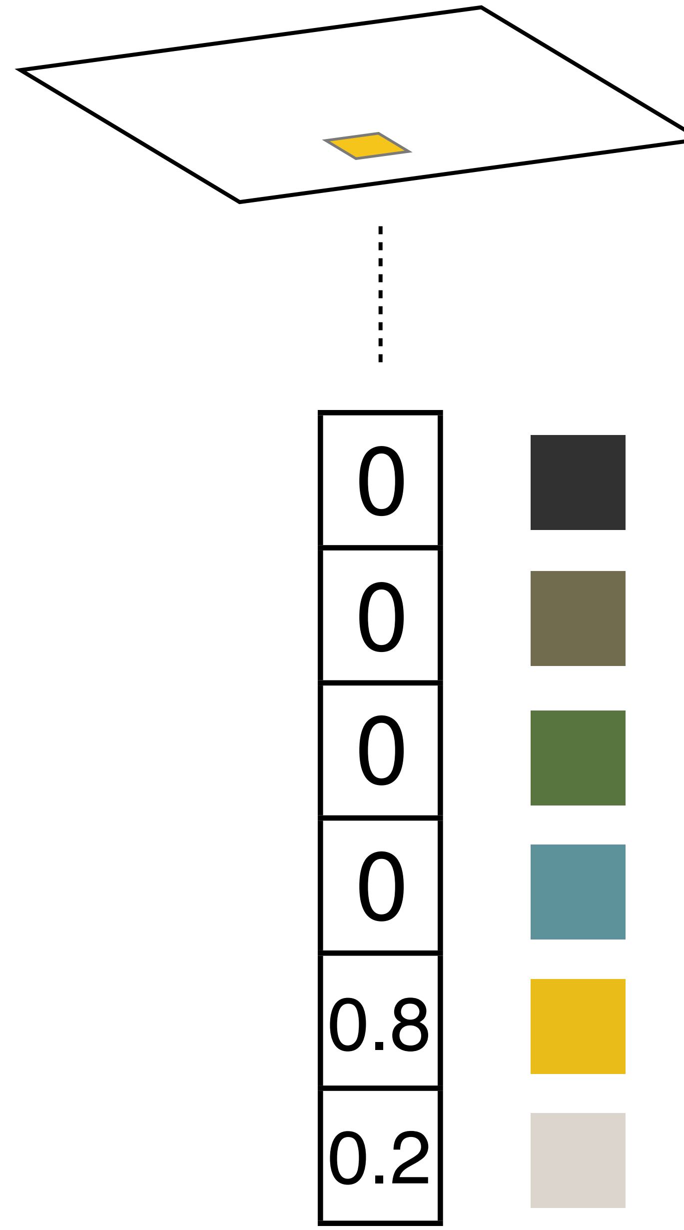
Image segmentation / Potts problem



assignment vector

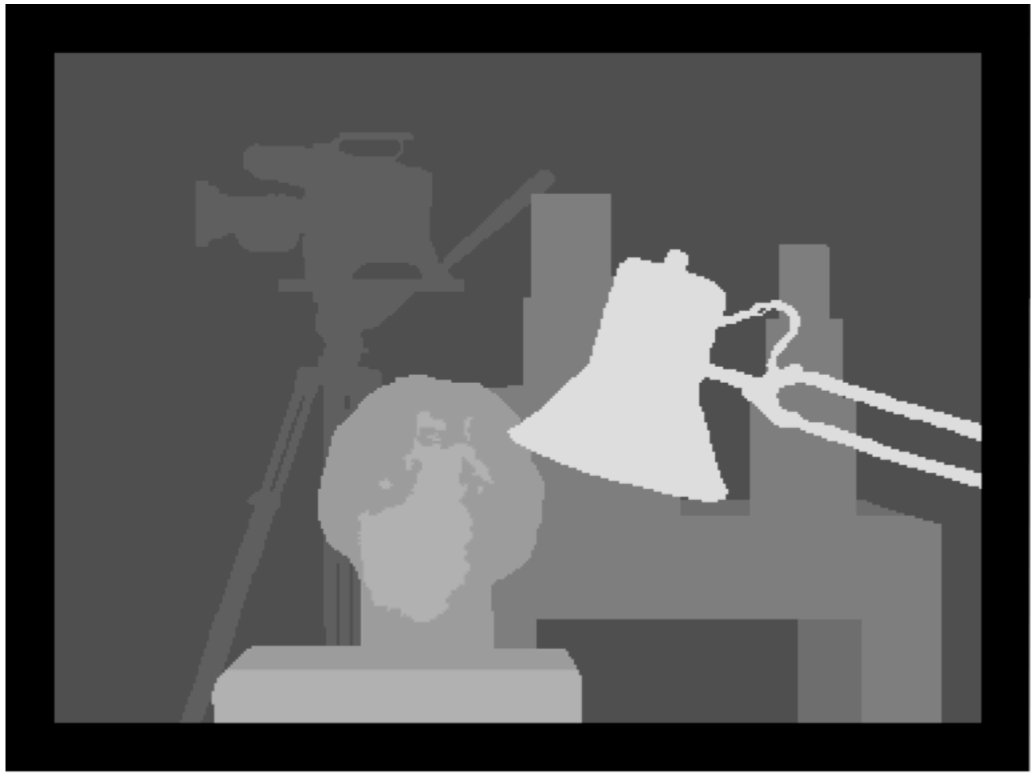
Image segmentation / Potts problem





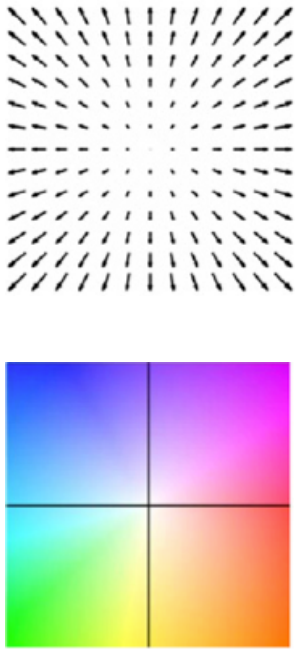
after relaxation: proportions

Other labeling problems



http://vision.middlebury.edu/stereo/eval/newEval/tsukuba/

Other labeling problems

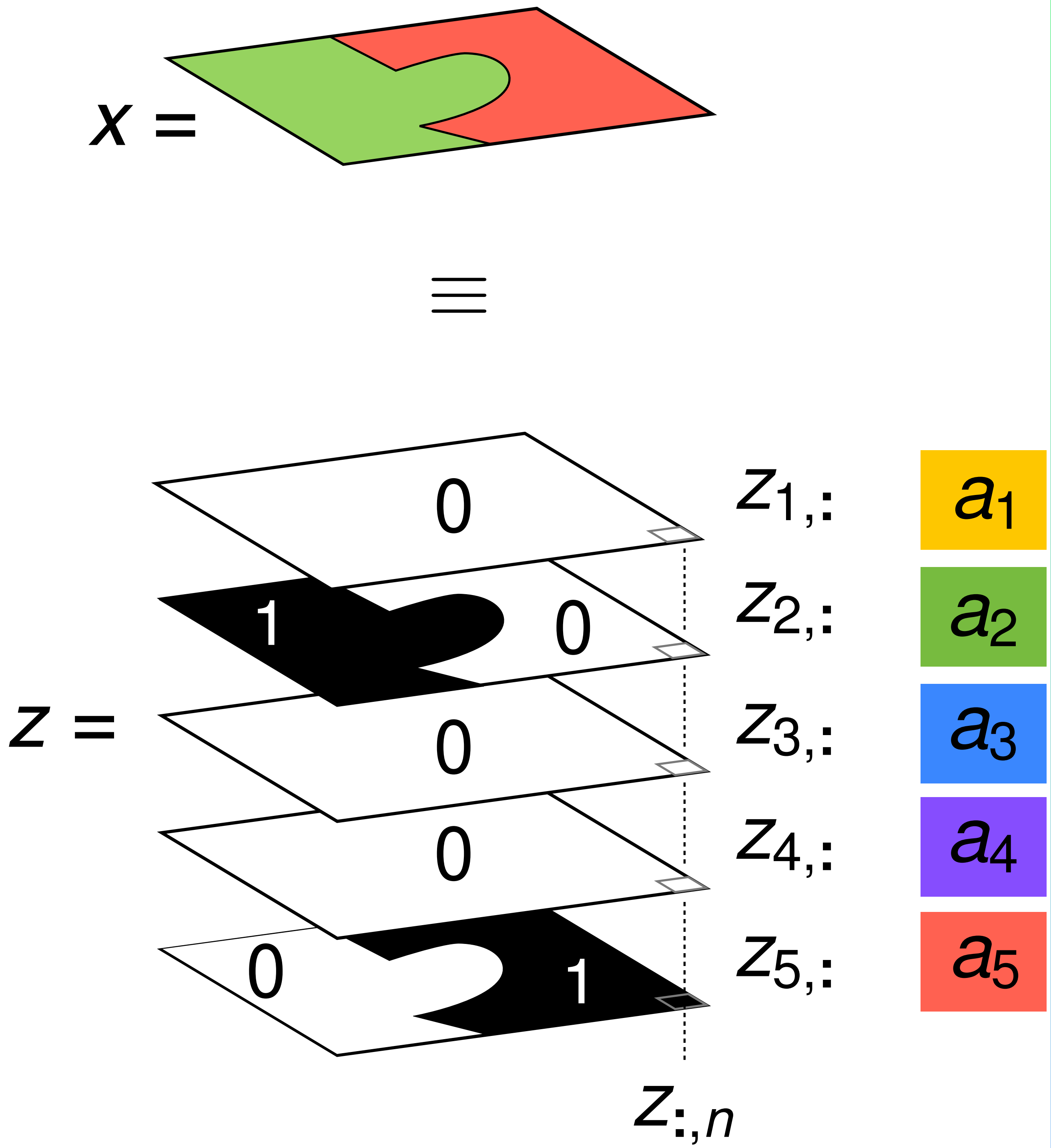


[D. Cremers et al.]

Convexified Potts problem

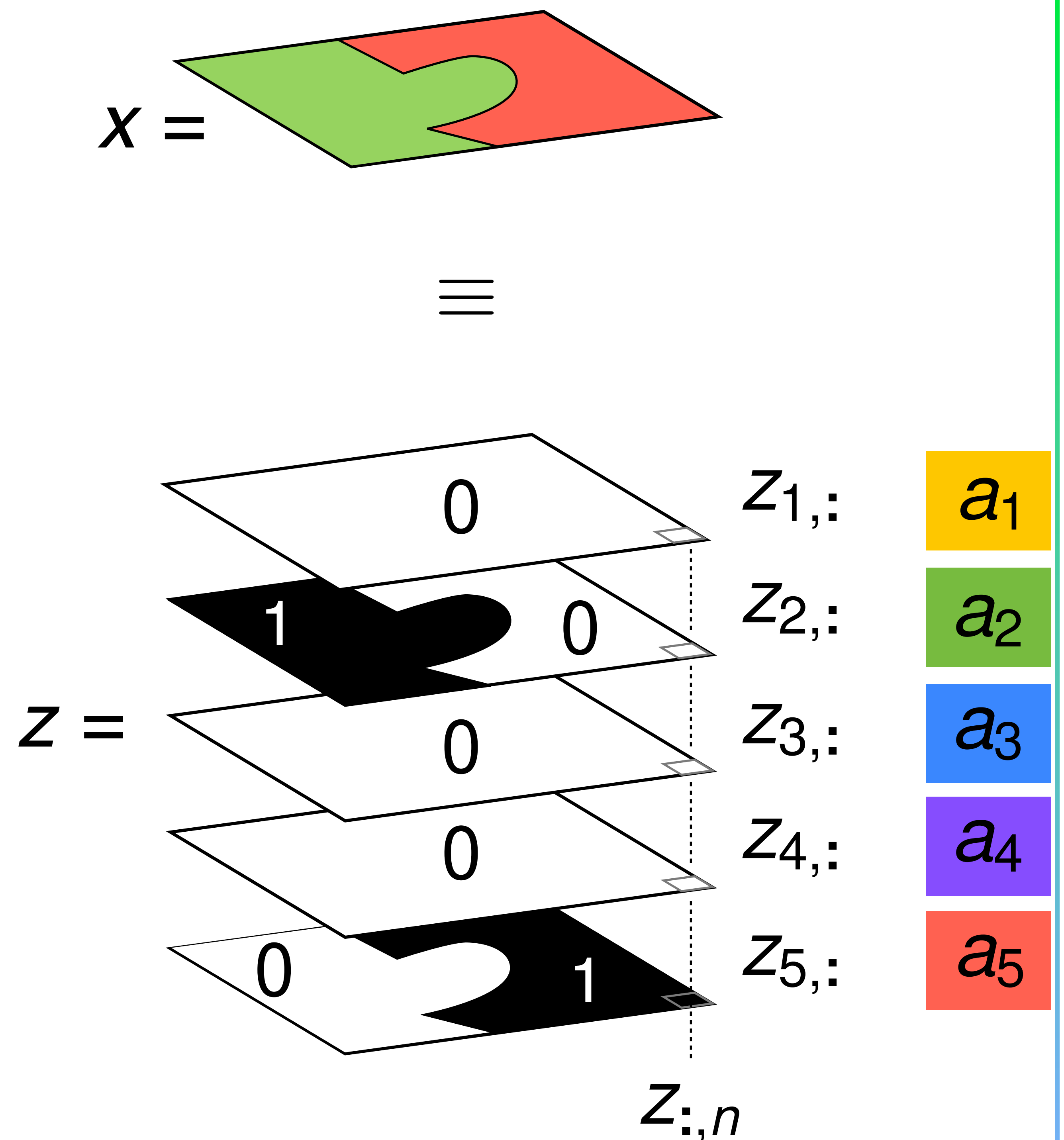
Finding x is equivalent to finding the *assignment array* z

☞ reformulate the initial problem with respect to z



Convexified Potts problem

Finding x is equivalent to finding the *assignment array* z

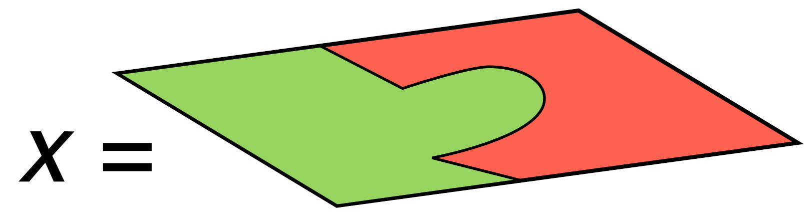


3 ingredients:

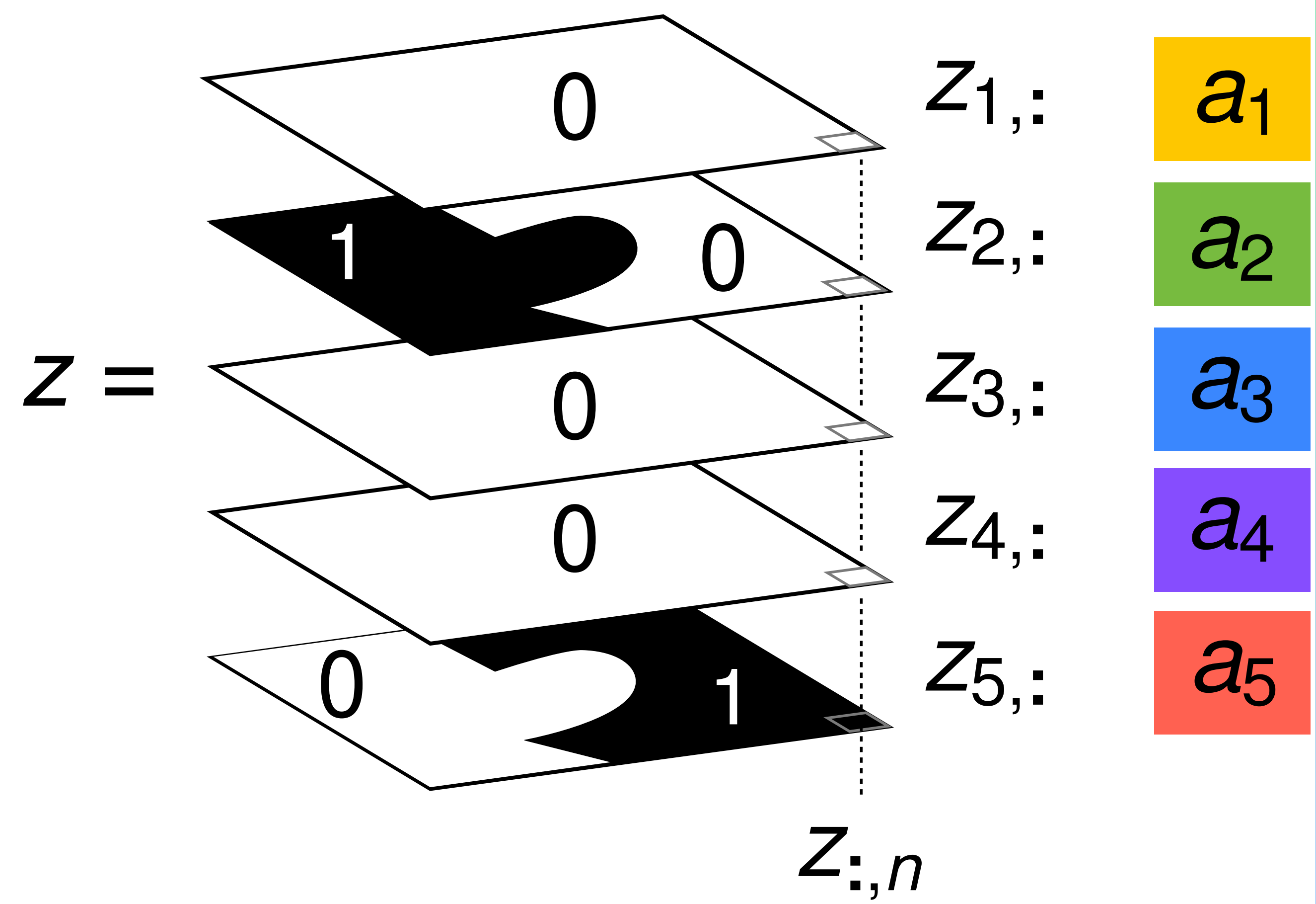
- $z_{:,n} \in \Delta, \forall n$

Convexified Potts problem

Finding x is equivalent to finding the *assignment array* z



$\equiv$

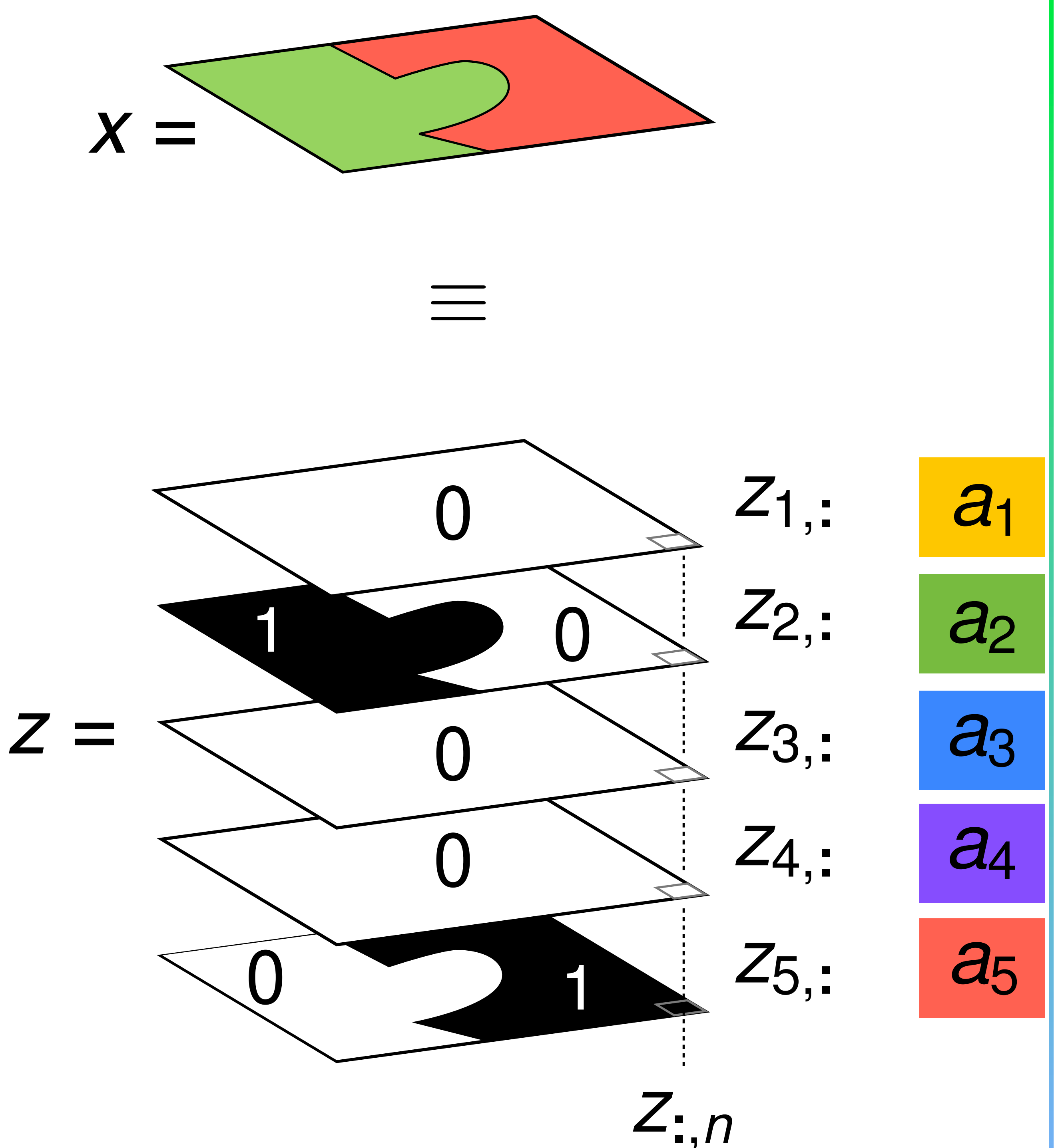


3 ingredients:

- $z_{:,n} \in \Delta, \forall n$
- loss term $\langle z, c\rangle$ where $c_{q,n}$ = cost of assigning label a_q to pixel of index n.

Convexified Potts problem

Finding x is equivalent to finding the *assignment array z*

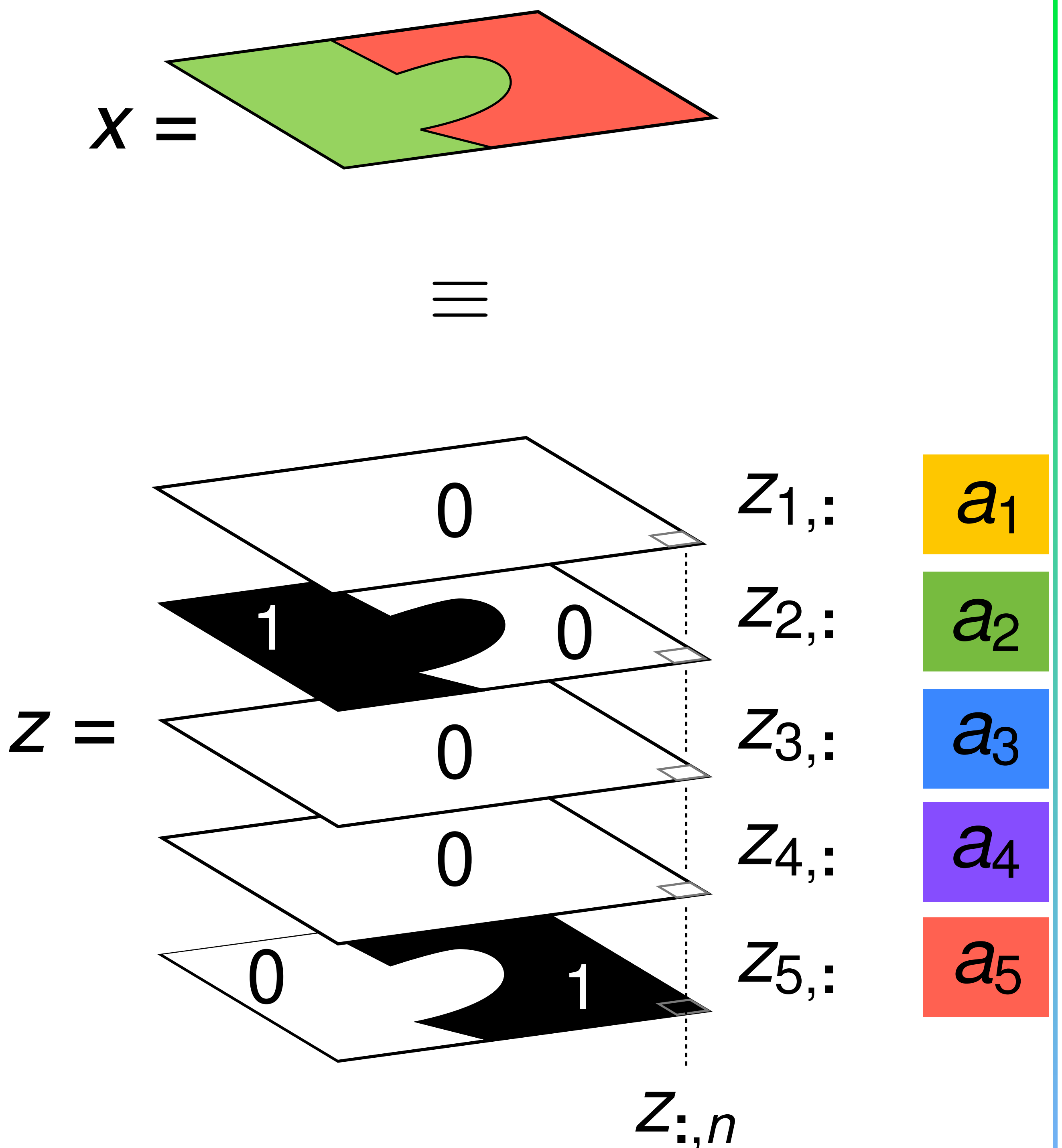


☞ 3 ingredients:

- $z_{:,n} \in \Delta, \forall n$
- loss term $\langle z, c \rangle$
  e.g. $c_{q,n} = \|y_n - a_q\|^2$

Convexified Potts problem

Finding x is equivalent to finding the *assignment array* z



☞ 3 ingredients:

- $z_{:,n} \in \Delta, \forall n$
- $\langle z, c \rangle$
- coarea formula: $\text{per}(\Omega_q) = \text{TV}(z_{q,:})$

Choice of the TV

isotropic TV

proposed TV

L. C., “Discrete total variation: New definition and minimization,” SIIMS, 2017.

Projection onto the simplex

Fast projection algorithms: L. Condat, "Fast projection onto the simplex and the l1 ball," Math. Prog., 2016

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split the simplex constraint into nonnegativity and sum to one

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$$r \in \mathbb{R}^{Q-1} \quad \text{s.t.} \quad 0 \leq r_1 \leq \cdots \leq r_{Q-1} \leq 1$$

differentiate

$$s_k = r_k - r_{k-1}$$

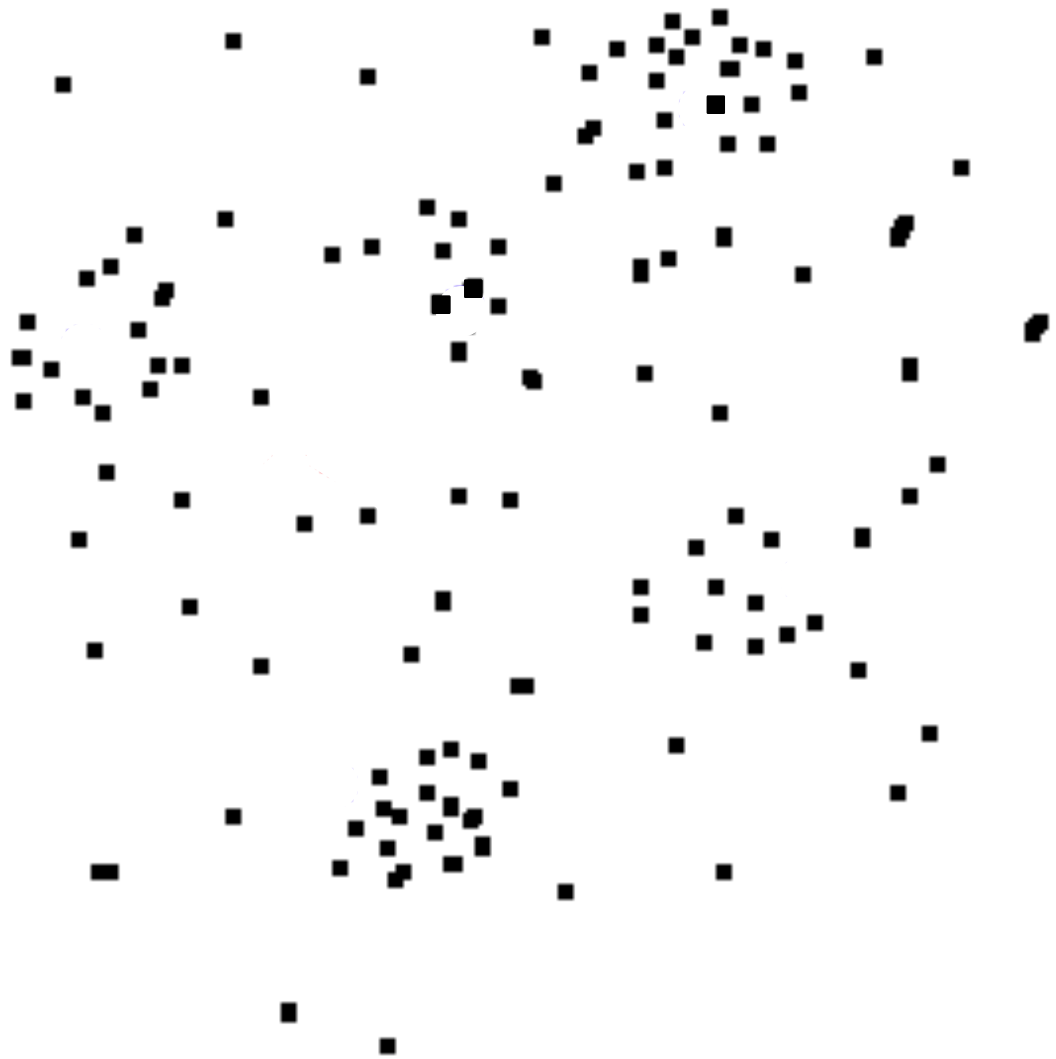
integrate

$$s \in \Delta \quad \equiv \quad s_q \geq 0, \sum_{q=1}^{Q} s_q = 1$$

N. Pustelnik and L. C., "Proximity operator of a sum of functions; application to depth map estimation," IEEE SPL, 2017

L. C, “A Convex Approach to K-means Clustering and Image Segmentation,” EMMCVPR, 2017

The K-means problem



The K-means problem

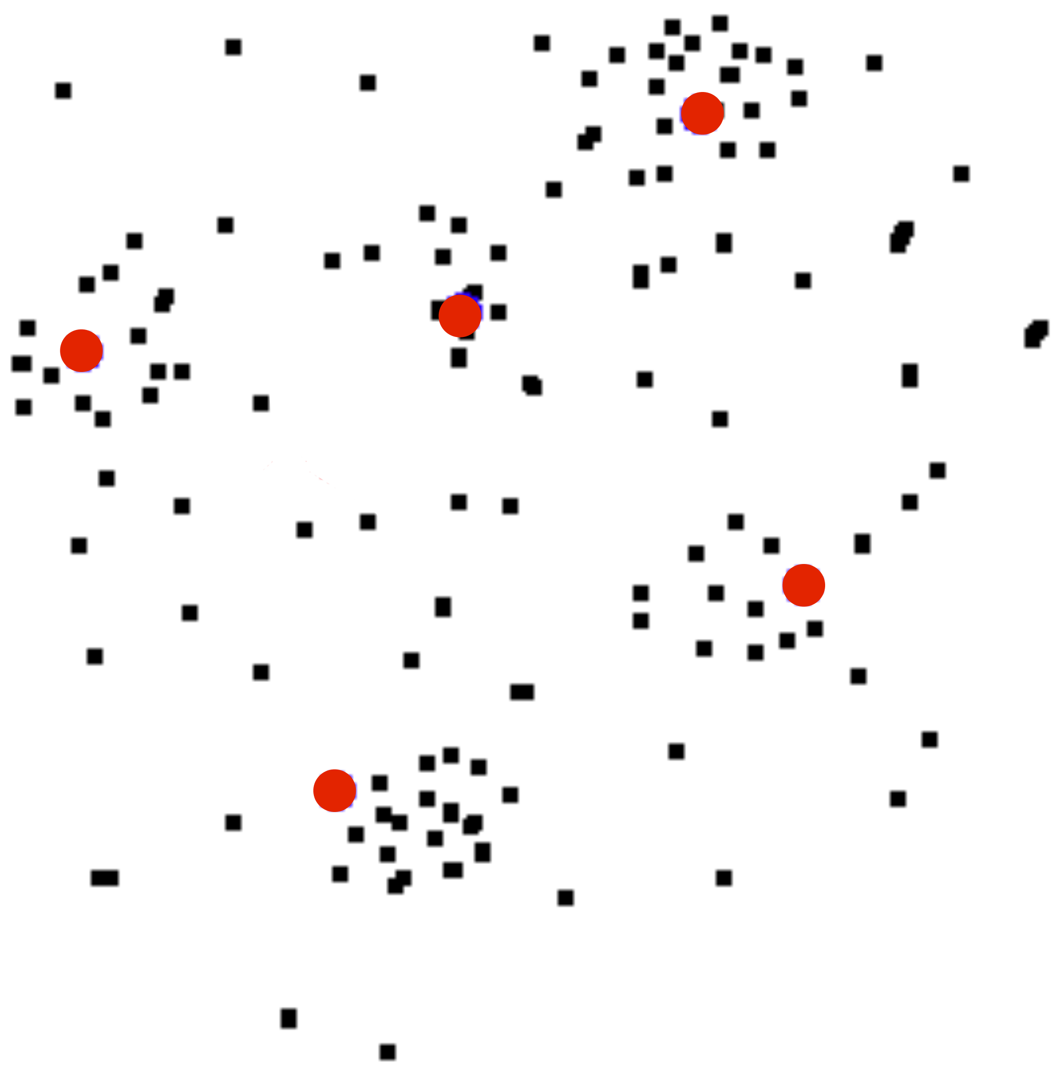


Image quantization

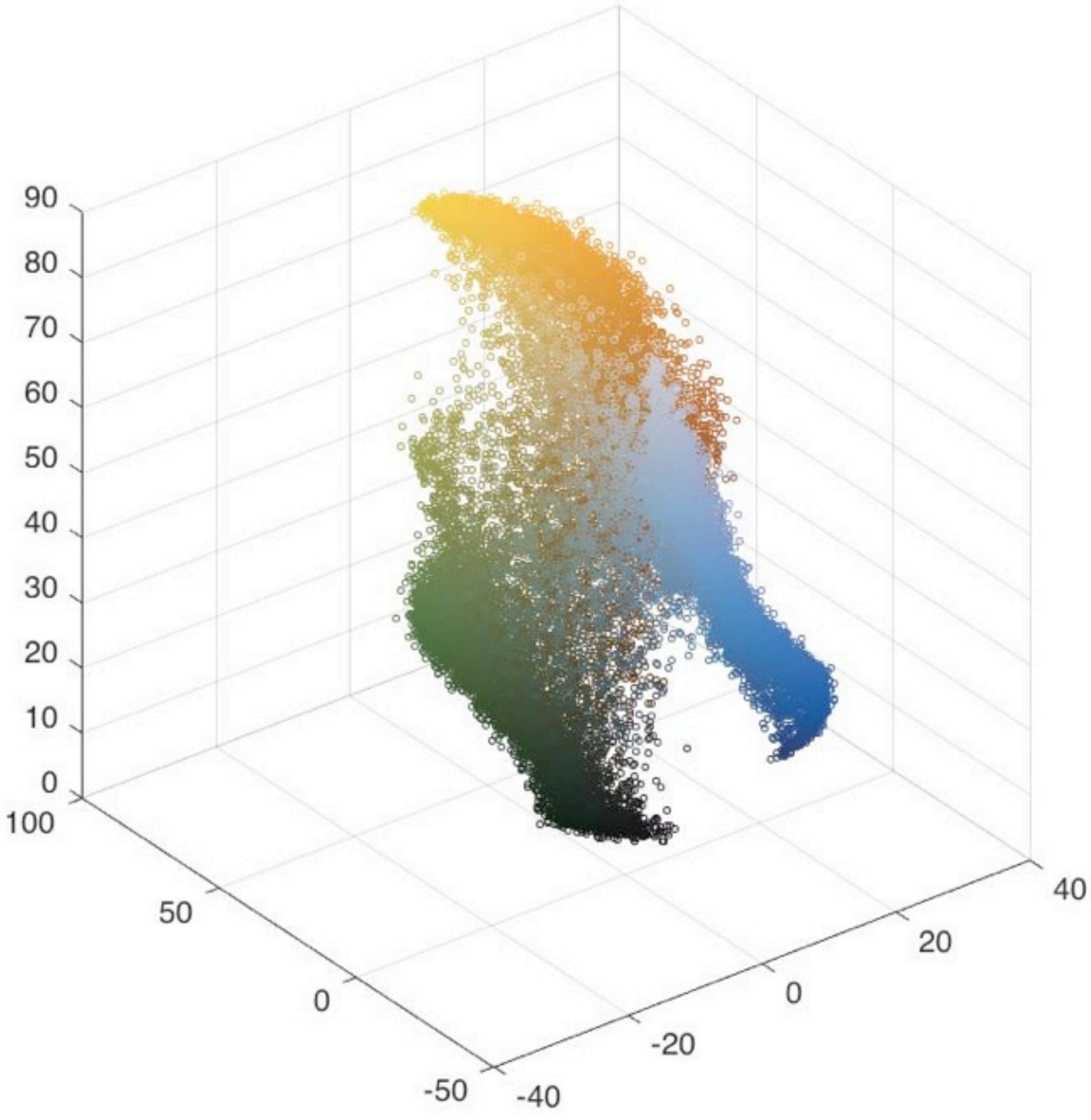


Image quantization

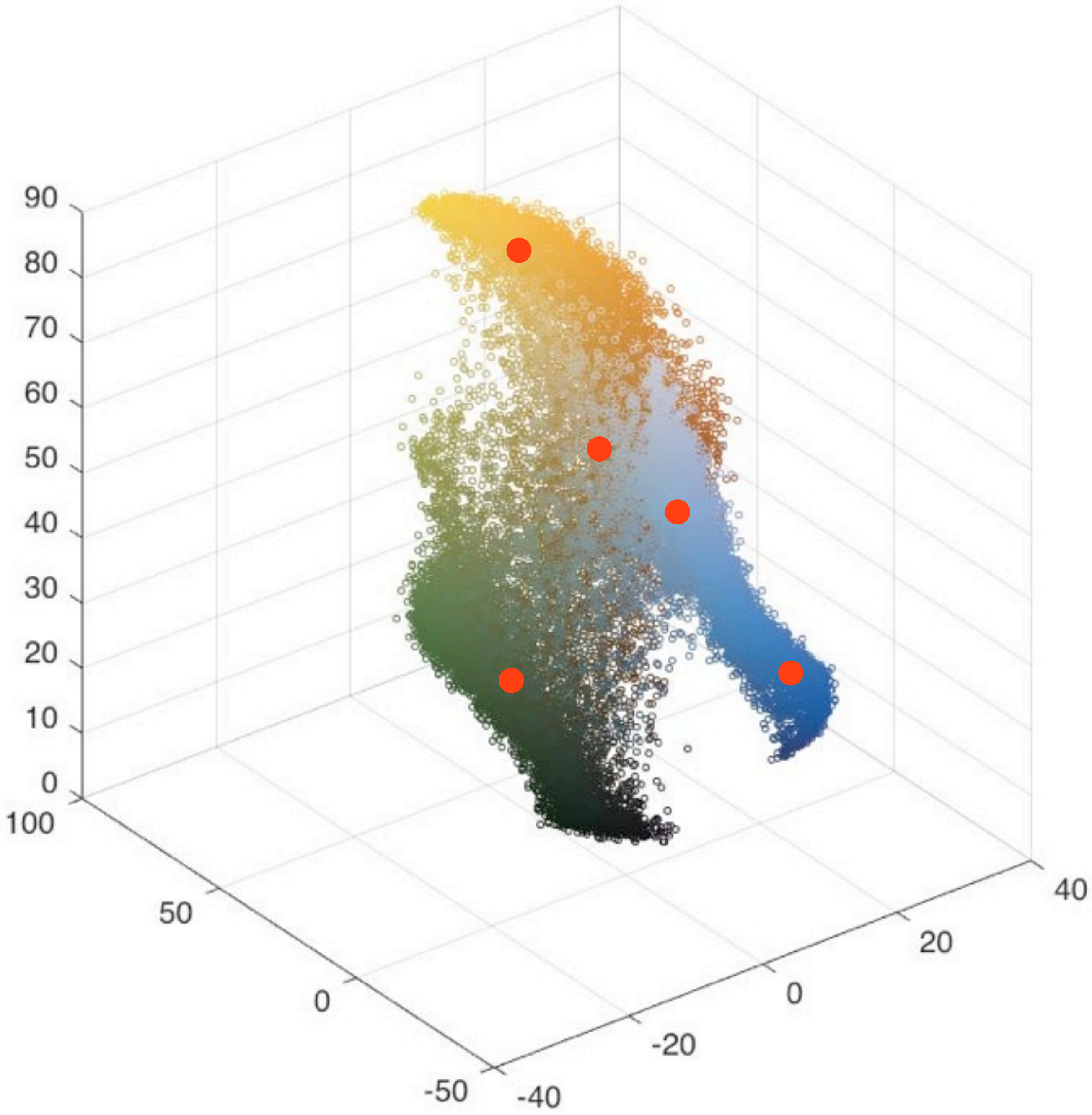
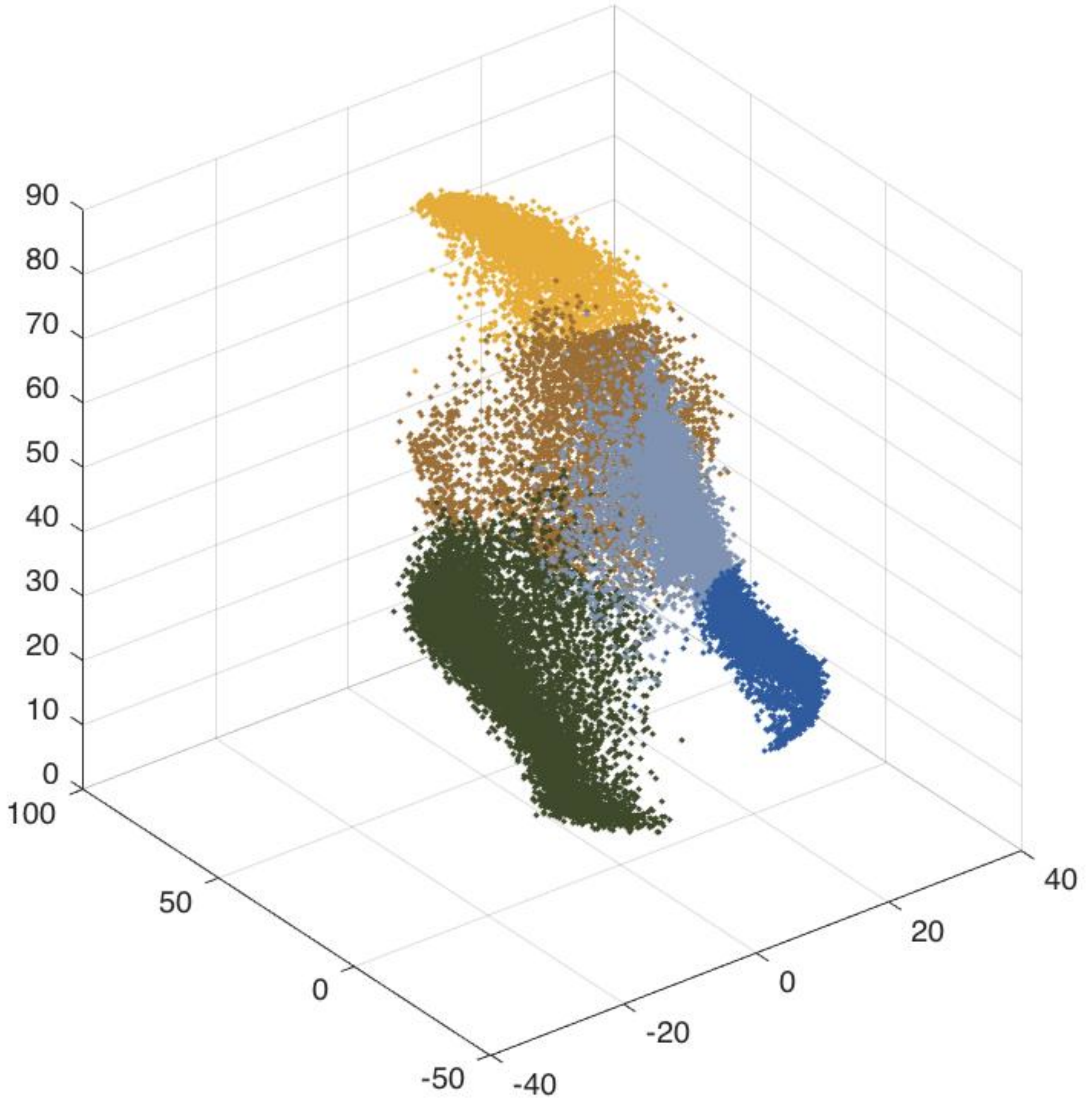


Image quantization



quantization vs. segmentation

Result with penalization of the region perimeter

Discrete search space

We discretize the search space of the centroids: they must belong to a finite set $\{a_q\}_{q=1}^{Q}$ of Q *candidates* of $\mathbb{R}^d$.

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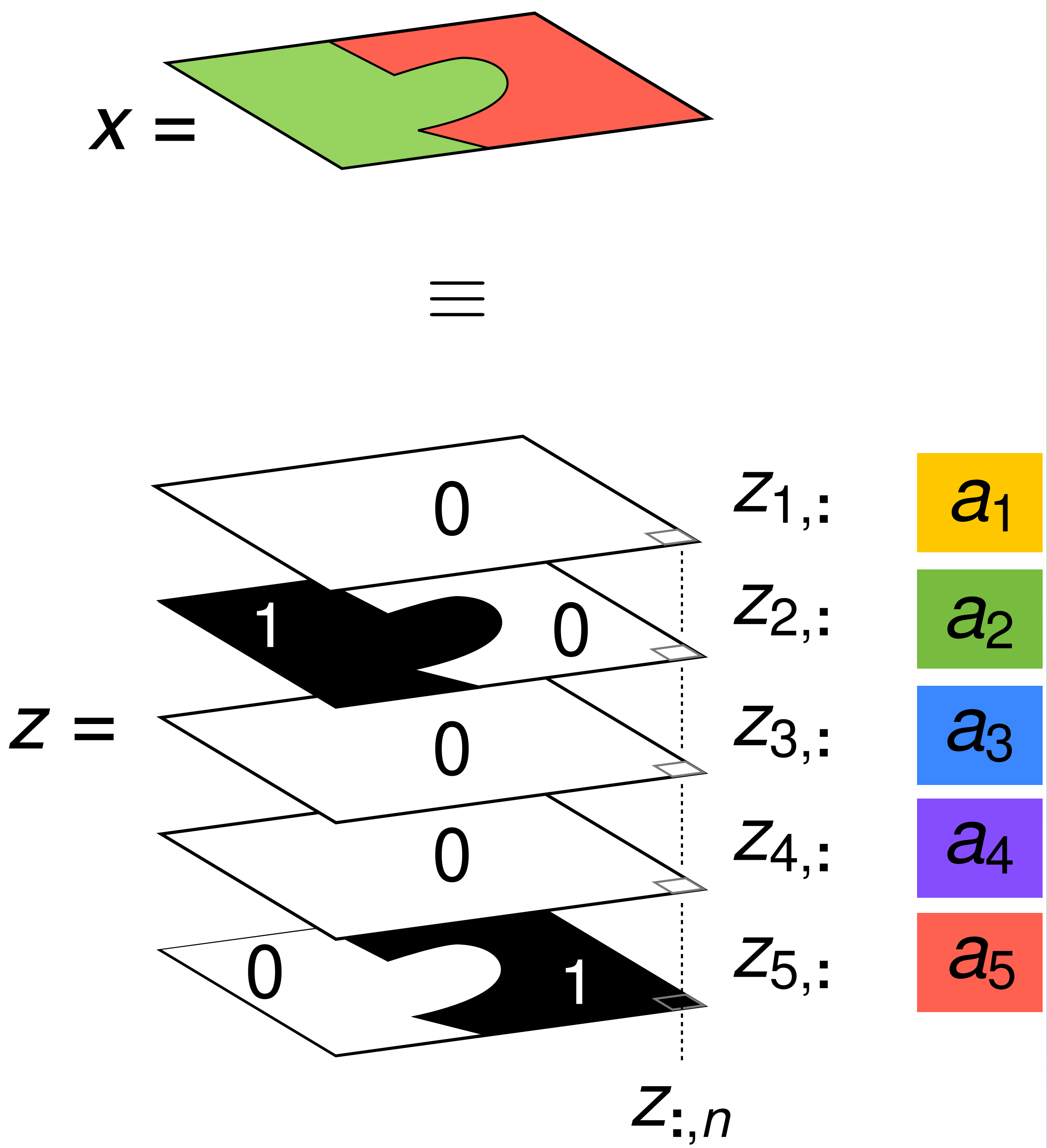
We discretize the search space of the centroids: they must belong to a finite set $\{a_q\}_{q=1}^{Q}$ of Q *candidates* of $\mathbb{R}^d$.

For color image quantization and segmentation, we choose the following *palette* of $Q = 279$ colors:

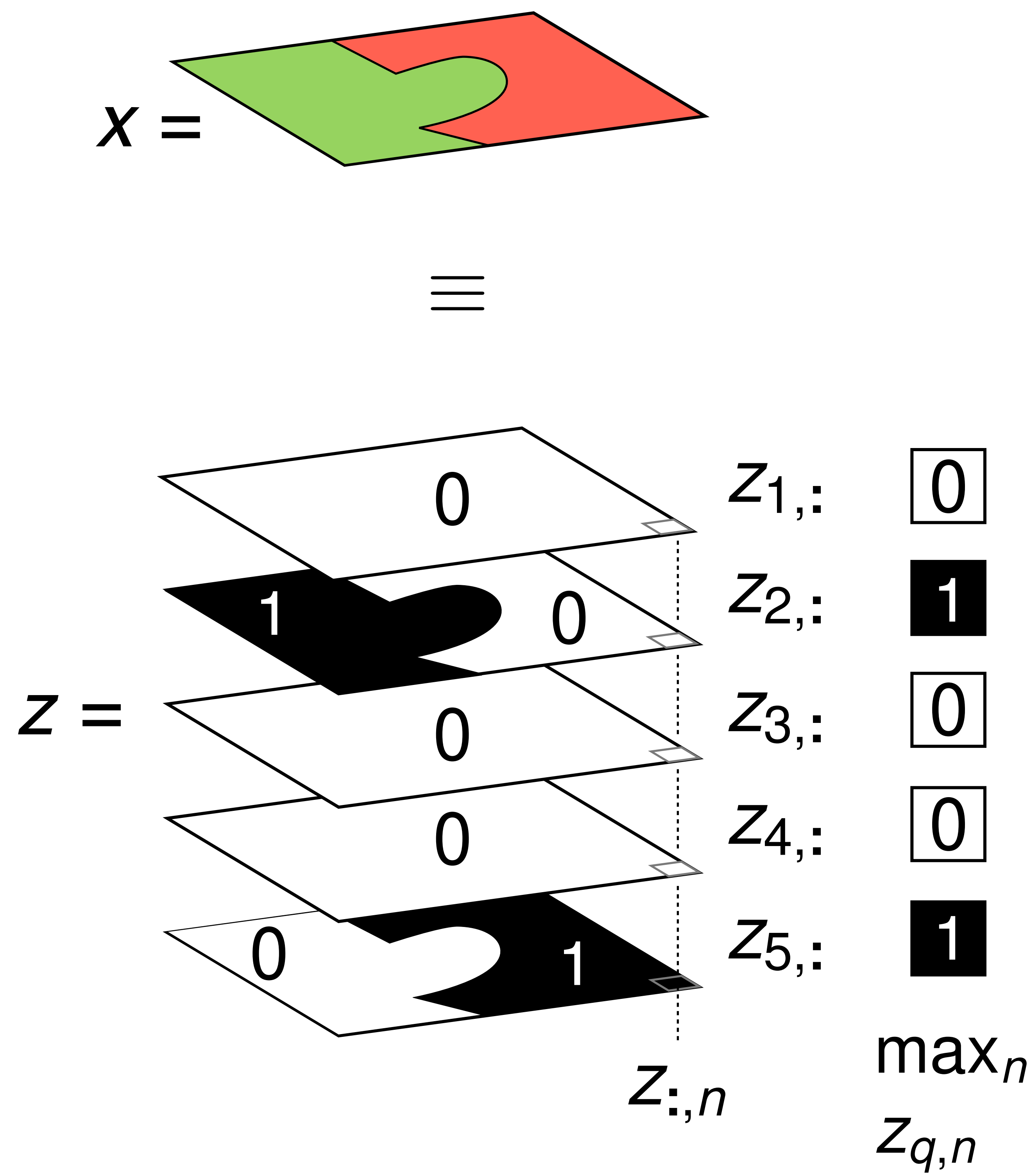
Lifted constraint of K classes



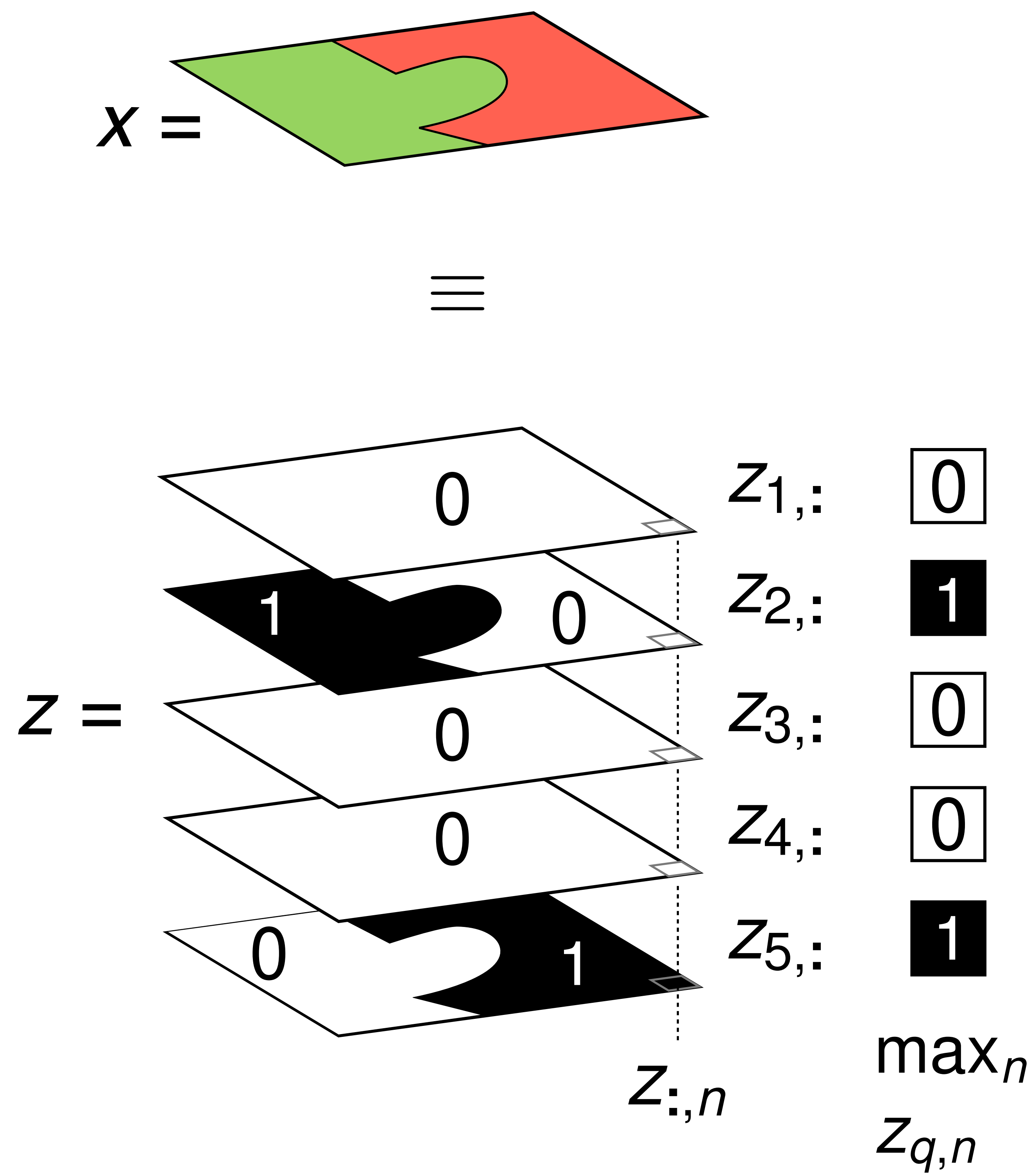
Lifted constraint of K classes

Nb. of active candidates is K $\equiv$

$$\sum_{q=1}^{Q} \max_{n\in\Omega} z_{q,n} \leq K$$



Lifted constraint of K classes



Nb. of active candidates is K $\equiv$

$$\sum_{q=1}^{Q} \max_{n\in\Omega} z_{q,n} \leq K$$

Projection on the $\ell_{1,\infty}$ ball: code on my webpage

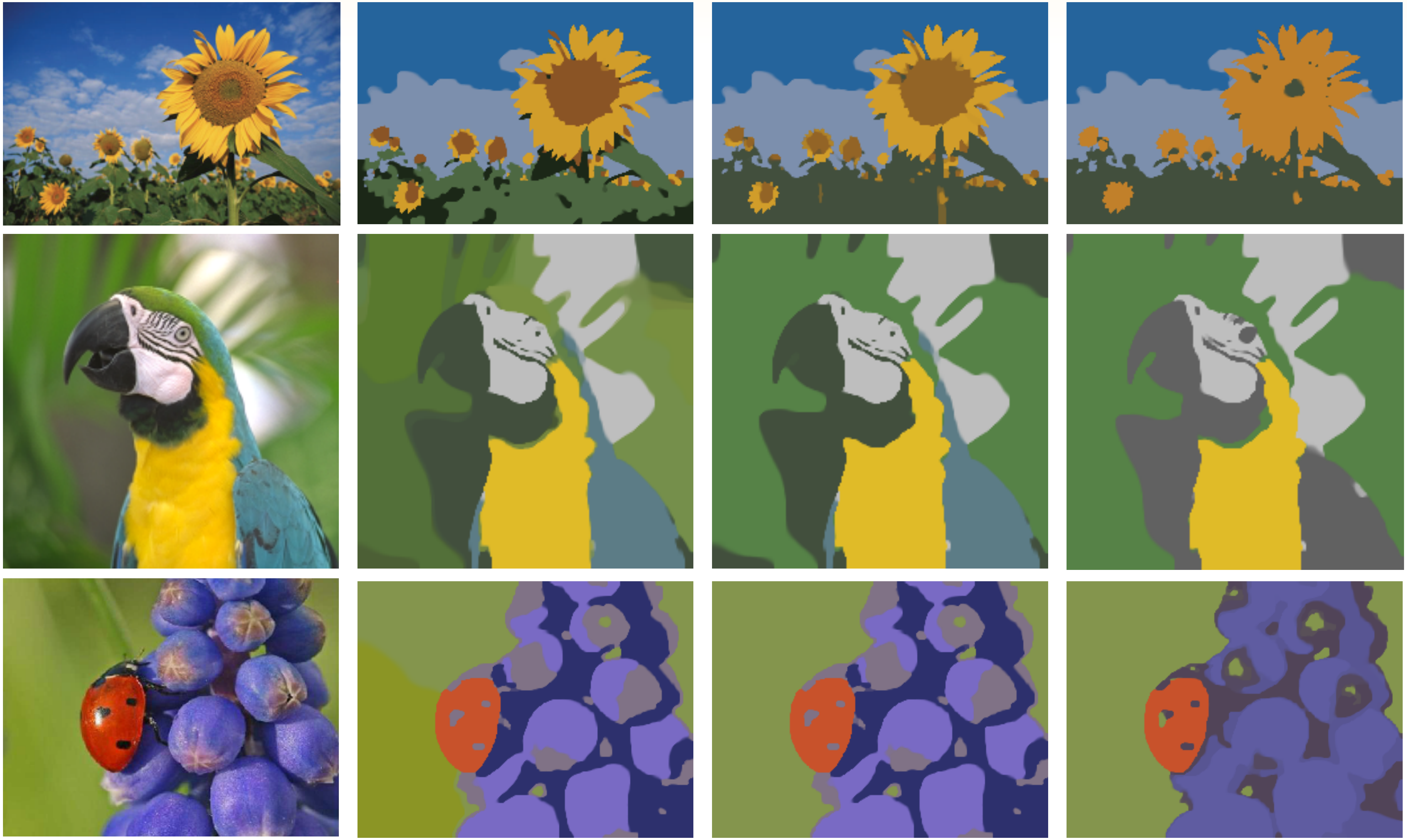
K-colors image segmentation

$$\underset{z\in\mathbb{R}^{M\times\Omega}}{\text{minimize}}\ \langle z, w\rangle + \lambda \sum_{q=1}^{Q} \mathrm{TV}(z_{q,:})$$

s.t. $z_{:,n} \in \Delta,\ \forall n \in \Omega$, and

$\sum_{q=1}^{Q} \max_{n\in\Omega}\ z_{q,n} \leq K$

Segmentation results



$K = 6$ $K = 5$ $K = 4$

Summary

Lifting: generic principle to formulate convex relaxations

Applications beyond labeling:

L. C. et al., “A convex lifting approach to image phase unwrapping,” 2019

What’s next?

- Try fast LP or ILP solvers
- Approaches without discretization

  L. C., “Atomic norm minimization for decomposition into complex exponentials,” preprint, 2018.

Thank you!