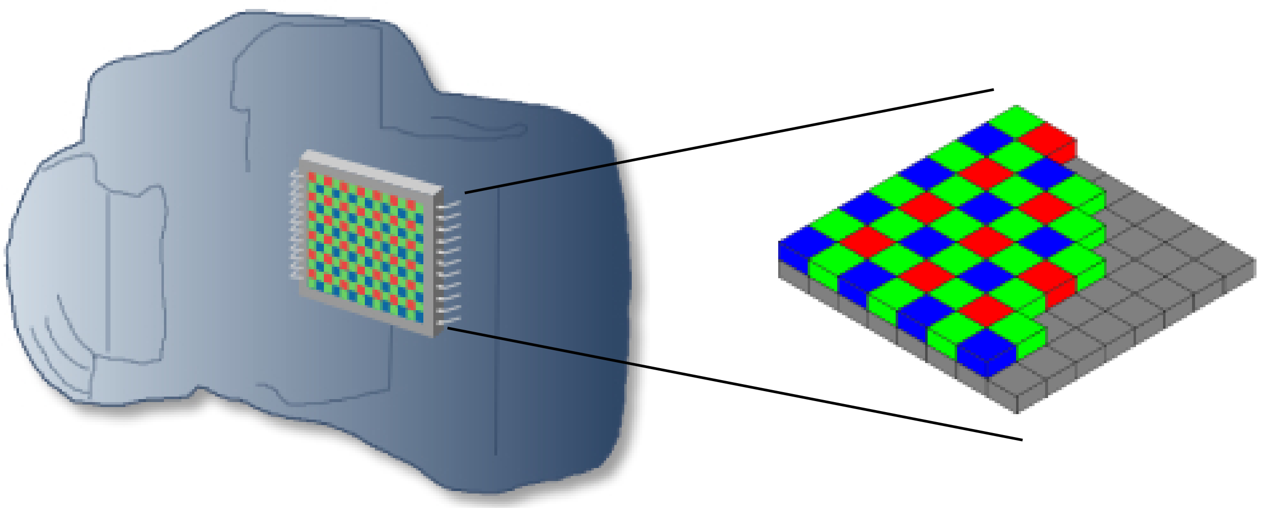
Models and methods for the acquisition of color images

Laurent Condat

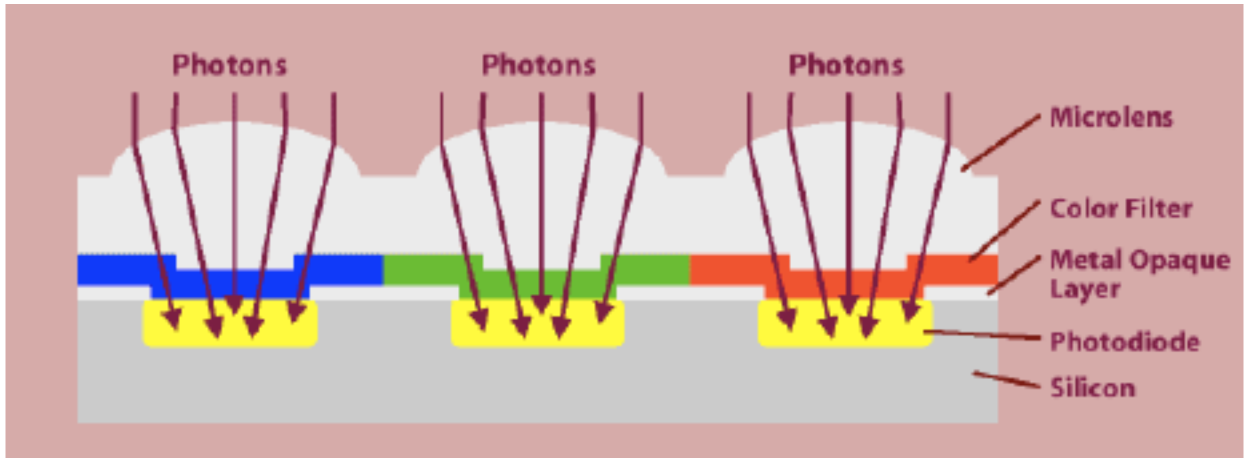
CNRS research fellow at GIPSA-lab, Grenoble, France

2013

Color image acquisition with a single sensor

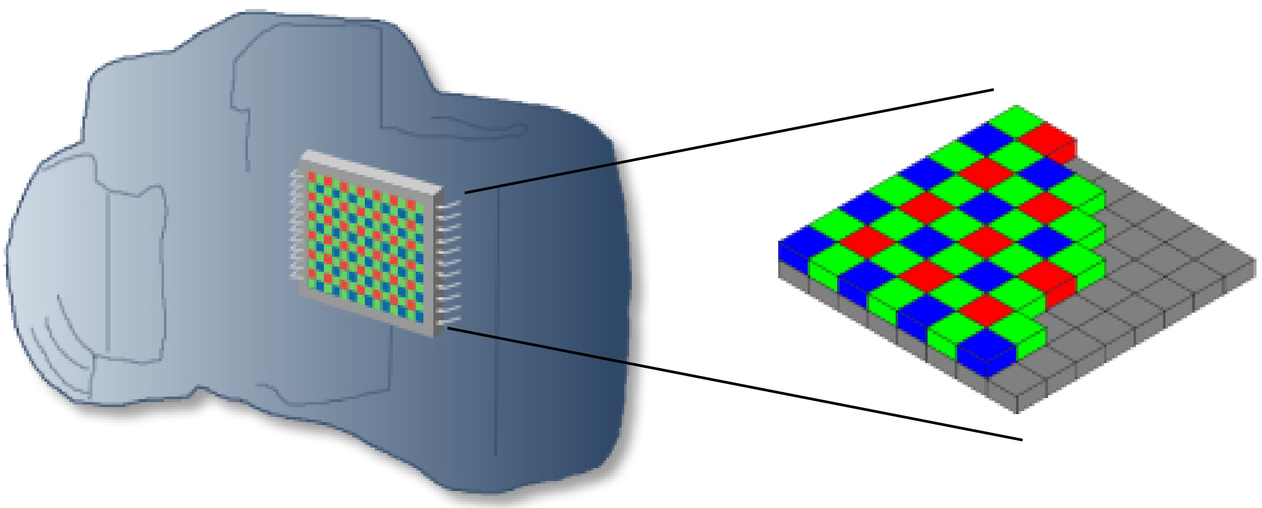


A (Bayer) color filter array (CFA) is overlaid on the sensor



what you see

Color image acquisition with a single sensor



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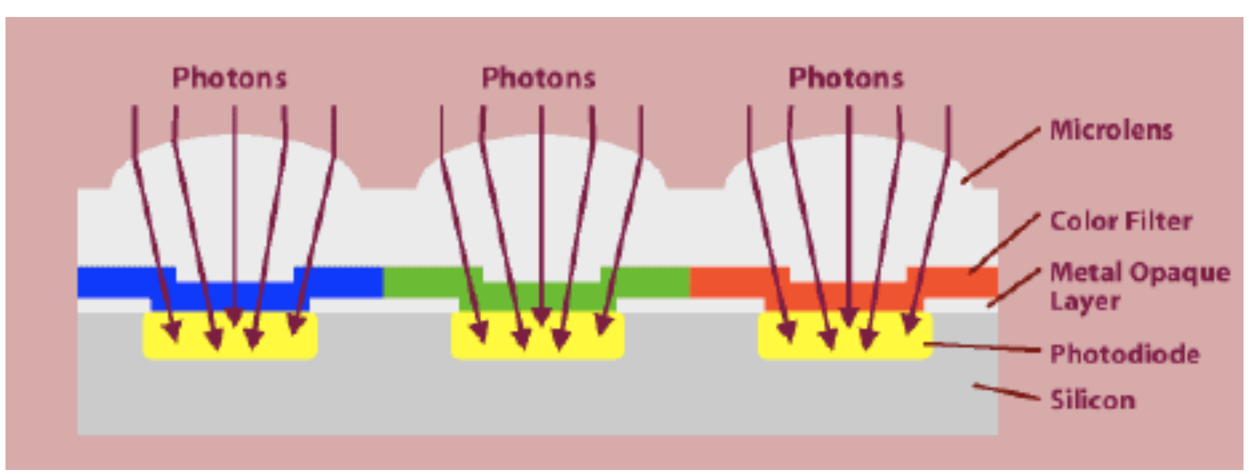
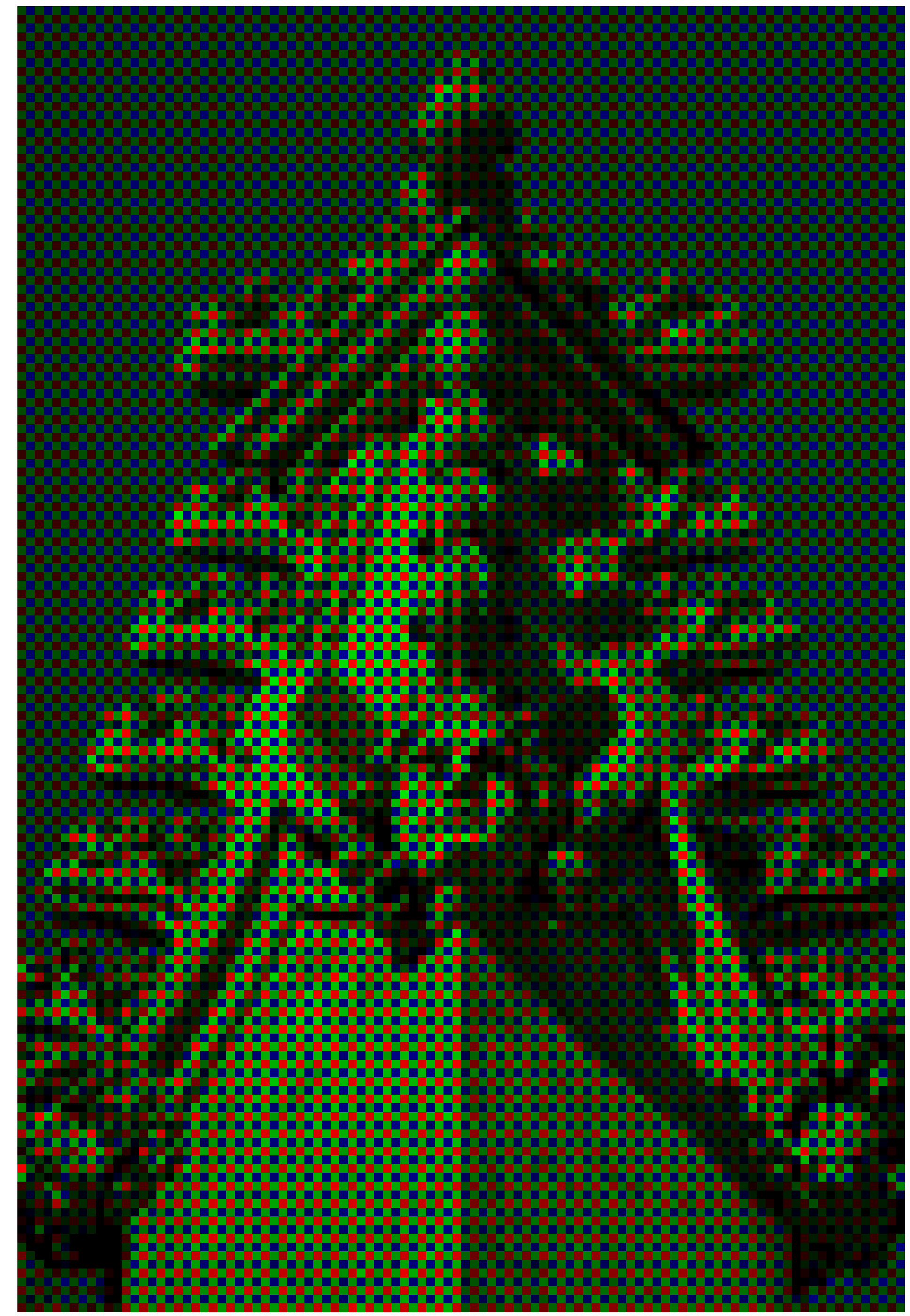
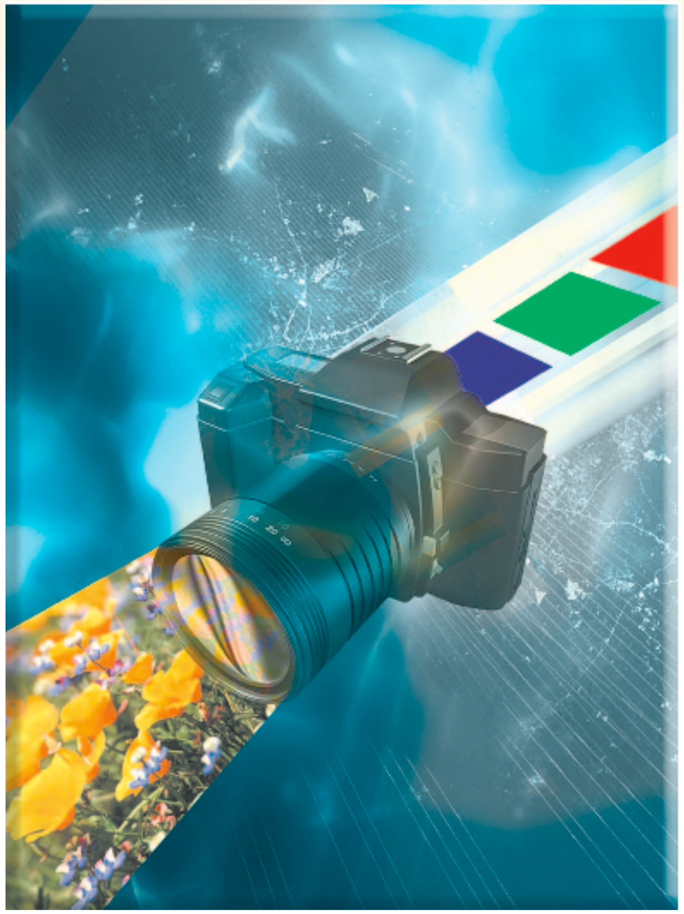




what your camera sees

Outline

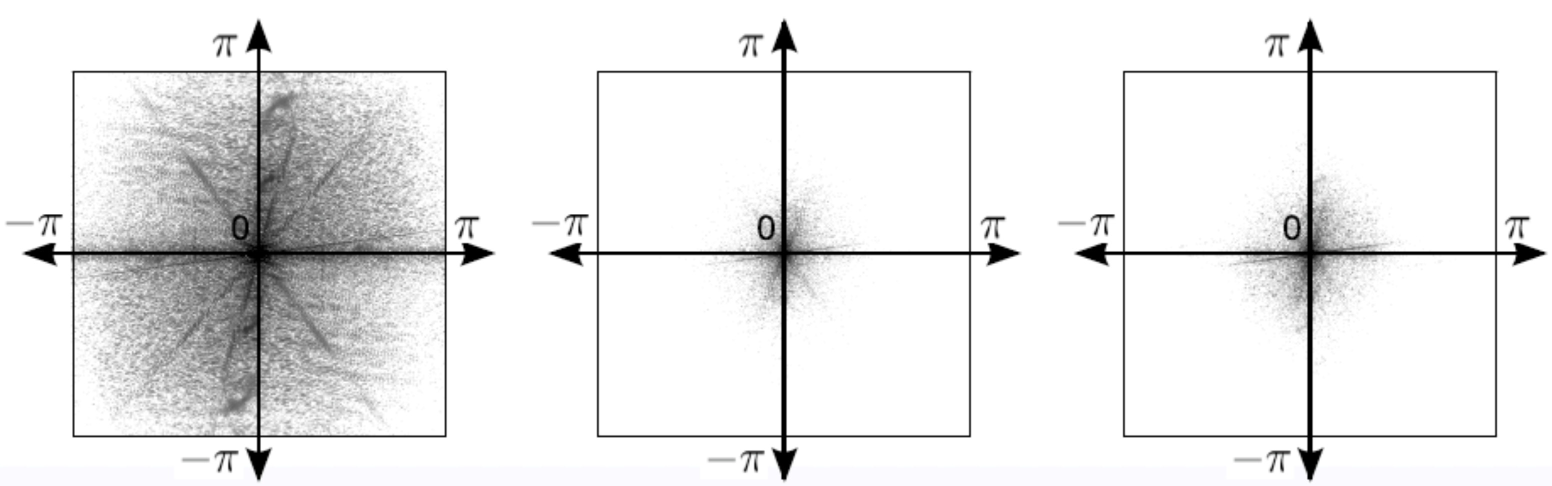


- The challenge of demosaicking
- An image formation model
- «Denoisaicking» methods
- New robust CFAs

Luminance / chrominance basis

$$=$$

$$+$$

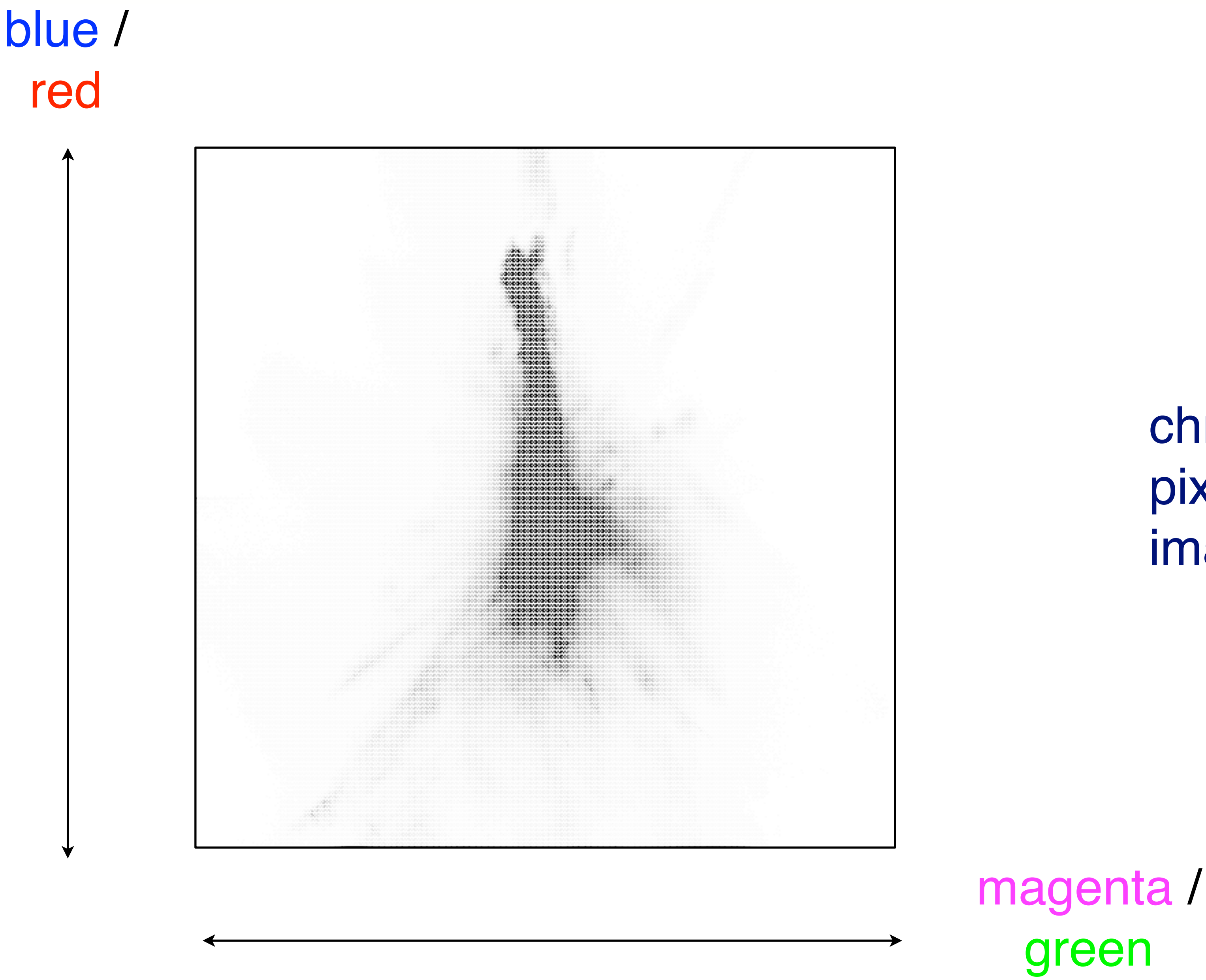


$$\mathbf{L} = \frac{1}{\sqrt{3}}[1,1,1]^{\mathrm{T}}$$

$$\mathbf{C}^{G/M} = \frac{1}{\sqrt{6}}[-1,2,-1]^{\mathrm{T}}$$

$$\mathbf{C}^{R/B} = \frac{1}{\sqrt{3}}[1,0,-1]^{\mathrm{T}}$$

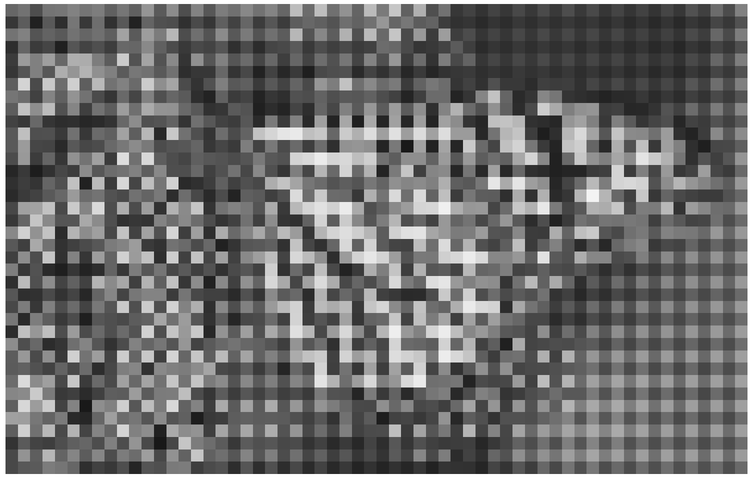
Chrominance of natural images



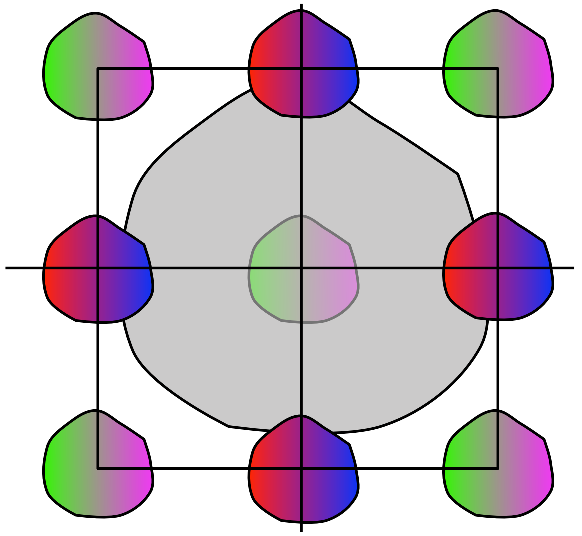
chrominance of all the pixels in a base of 150 images

Frequency interpretation of Bayer sampling

[Alleysson *et al.*, *IEEE TIP*, 2005]



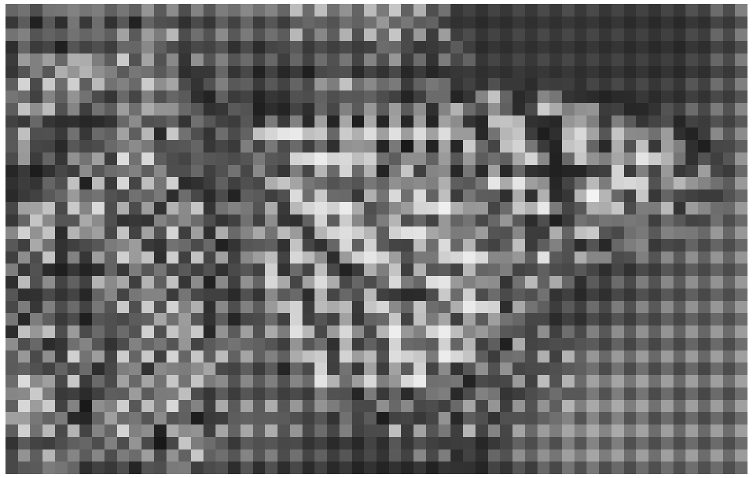
Fourier transform



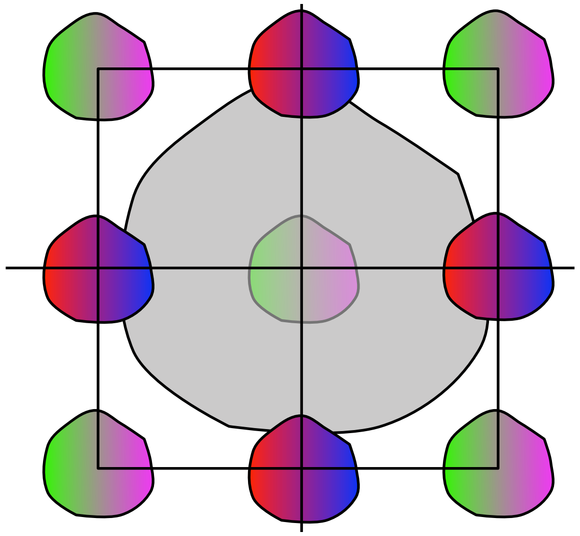
$$\hat{v}(\boldsymbol{\omega}) = \frac{1}{\sqrt{3}}\hat{u}^{L}(\boldsymbol{\omega}) + \frac{1}{\sqrt{24}}\hat{u}^{G/M}(\boldsymbol{\omega}) + \frac{\sqrt{6}}{4}\hat{u}^{G/M}(\boldsymbol{\omega} - [\pi,\pi]^{\mathrm{T}}) + \frac{\sqrt{2}}{4}\hat{u}^{R/B}(\boldsymbol{\omega} - [0,\pi]^{\mathrm{T}}) - \frac{\sqrt{2}}{4}\hat{u}^{R/B}(\boldsymbol{\omega} - [\pi,0]^{\mathrm{T}})$$

Frequency interpretation of Bayer sampling

[Alleysson *et al.*, *IEEE TIP*, 2005]



Fourier transform →



$$v[\mathbf{k}] = \frac{1}{\sqrt{3}} u^L[\mathbf{k}] + \frac{1}{\sqrt{24}} u^{G/M}[\mathbf{k}] + \frac{\sqrt{6}}{4}(-1)^{k_1+k_2} u^{G/M}[\mathbf{k}] + \frac{\sqrt{2}}{4}(-1)^{k_2} u^{R/B}[\mathbf{k}] - \frac{\sqrt{2}}{4}(-1)^{k_1} u^{R/B}[\mathbf{k}]$$

Linear demosaicking by frequency selection

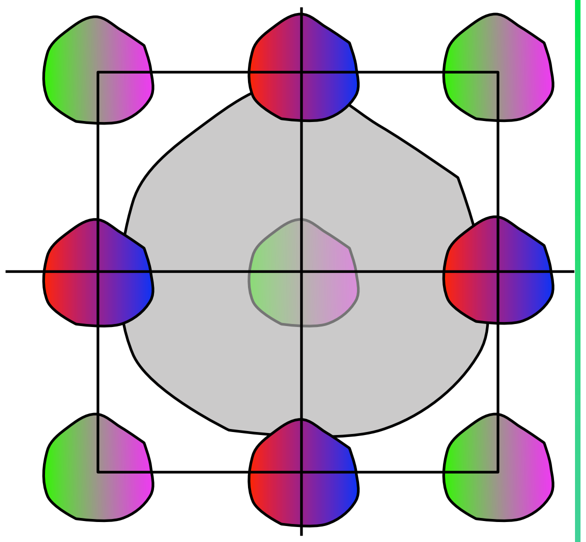
- Chrominance obtained by modulation + lowpass filtering

$$d^{G/M} = \frac{4}{\sqrt{6}} v_{\pi,\pi} * h_{G/M} \text{ where } v_{\pi,\pi}[\mathbf{k}] = (-1)^{k_1+k_2} v[\mathbf{k}]$$

$$d_H^{R/B} = -2\sqrt{2}\, v_{\pi,0} * h_{R/B} \text{ where } v_{\pi,0}[\mathbf{k}] = (-1)^{k_1} v[\mathbf{k}]$$

$$d_V^{R/B} = 2\sqrt{2}\, v_{0,\pi} * (h_{R/B})^{\mathrm{T}} \text{ where } v_{0,\pi}[\mathbf{k}] = (-1)^{k_2} v[\mathbf{k}]$$

$$d^{R/B} = \frac{1}{2}(d_H^{R/B} + d_V^{R/B})$$



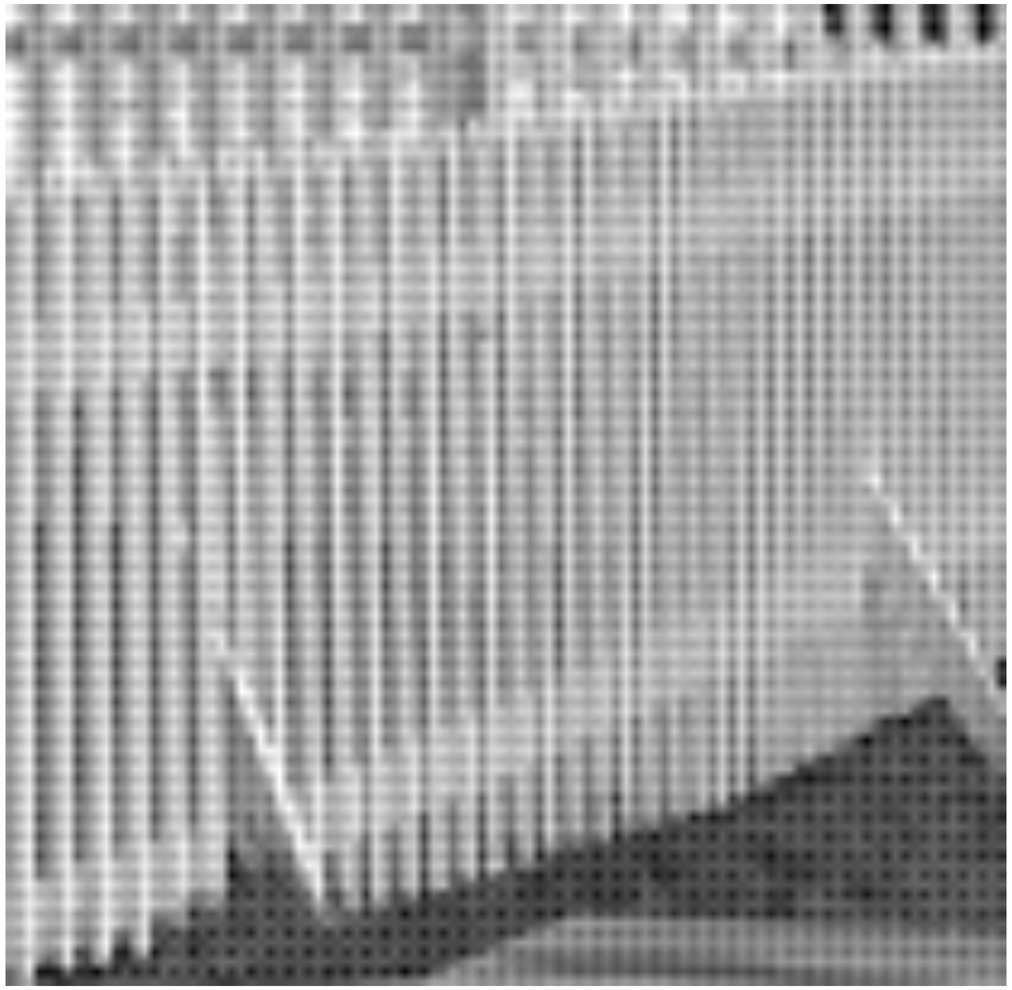
- Luminance as the residual

$$\frac{1}{\sqrt{3}} d^L = v[\mathbf{k}] - \left(\frac{1}{\sqrt{24}} + \frac{\sqrt{6}}{4}(-1)^{k_1+k_2}\right) d^{G/M}[\mathbf{k}] - \frac{\sqrt{2}}{4}\left((-1)^{k_2} - (-1)^{k_1}\right) d^{R/B}[\mathbf{k}]$$

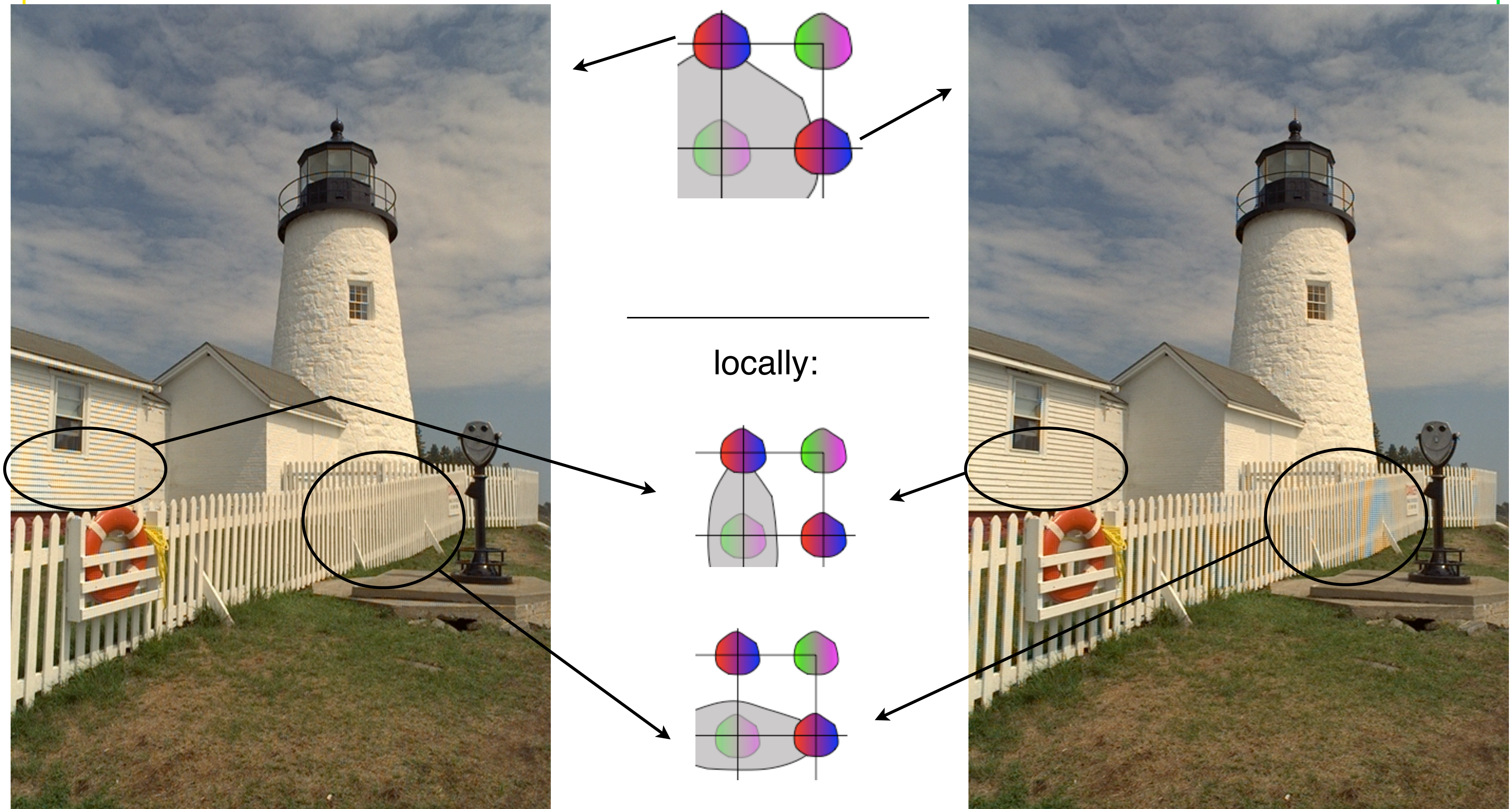
[Dubois, *IEEE SPL*, 2005]

Result

- Aliasing artifacts due to high-frequency content of luminance

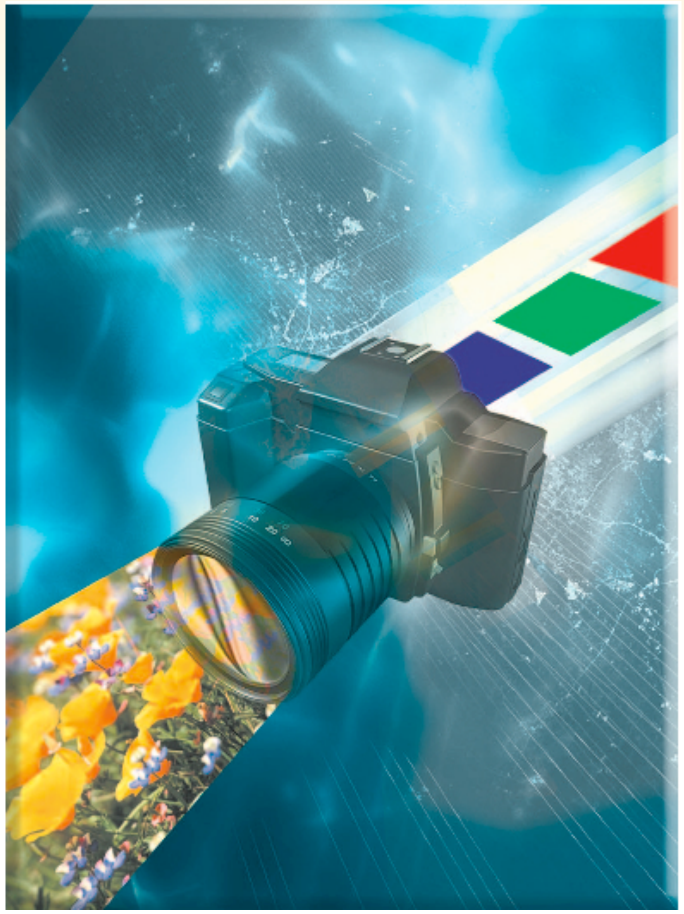


Redundancy of the blue/red chrominance information



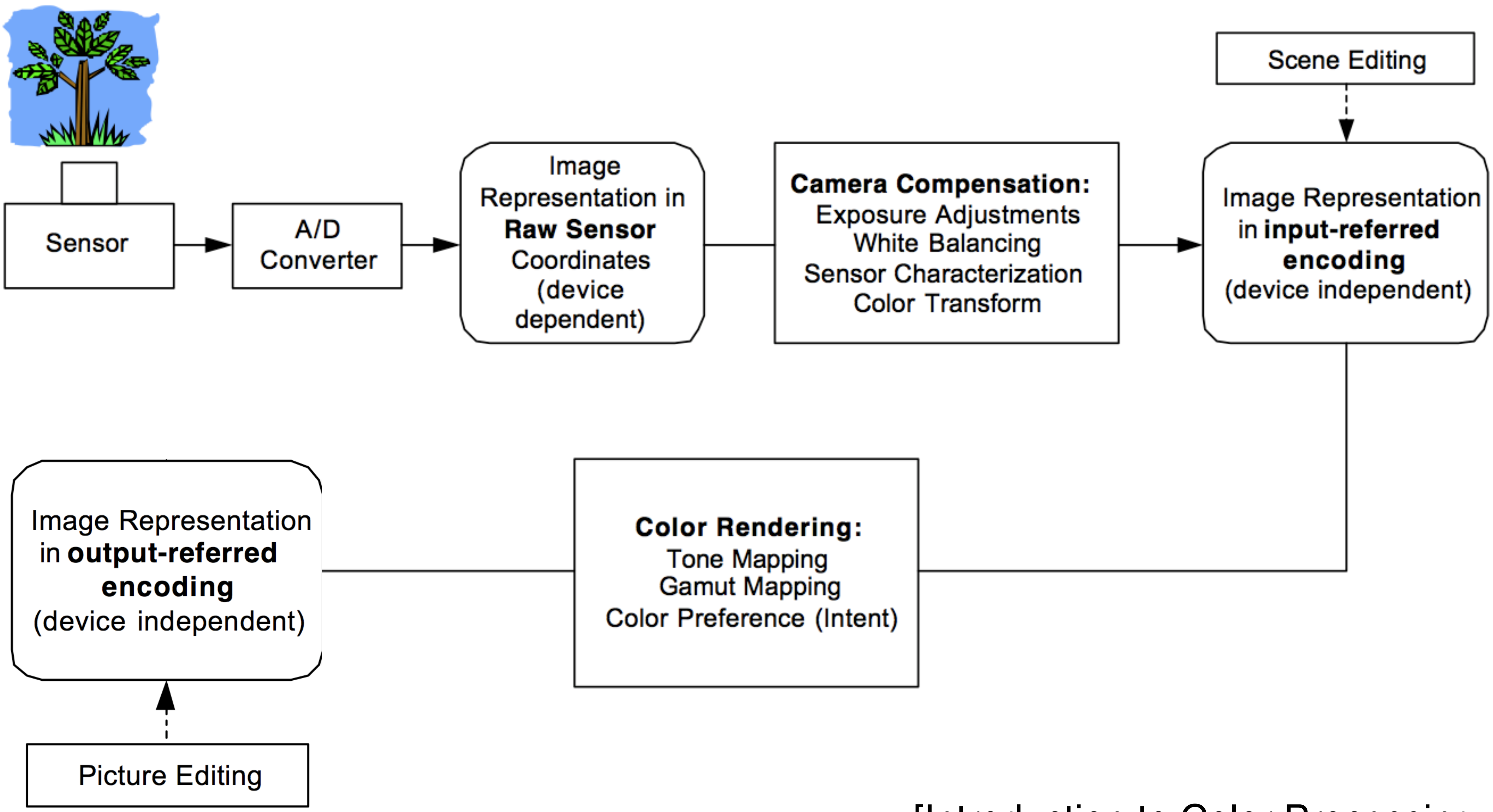
Adaptive demosaicking based on the structure tensor

Outline



- The challenge of demosaicking
- → An image formation model
- «Denoisaicking» methods
- New robust CFAs

The workflow of digital photography

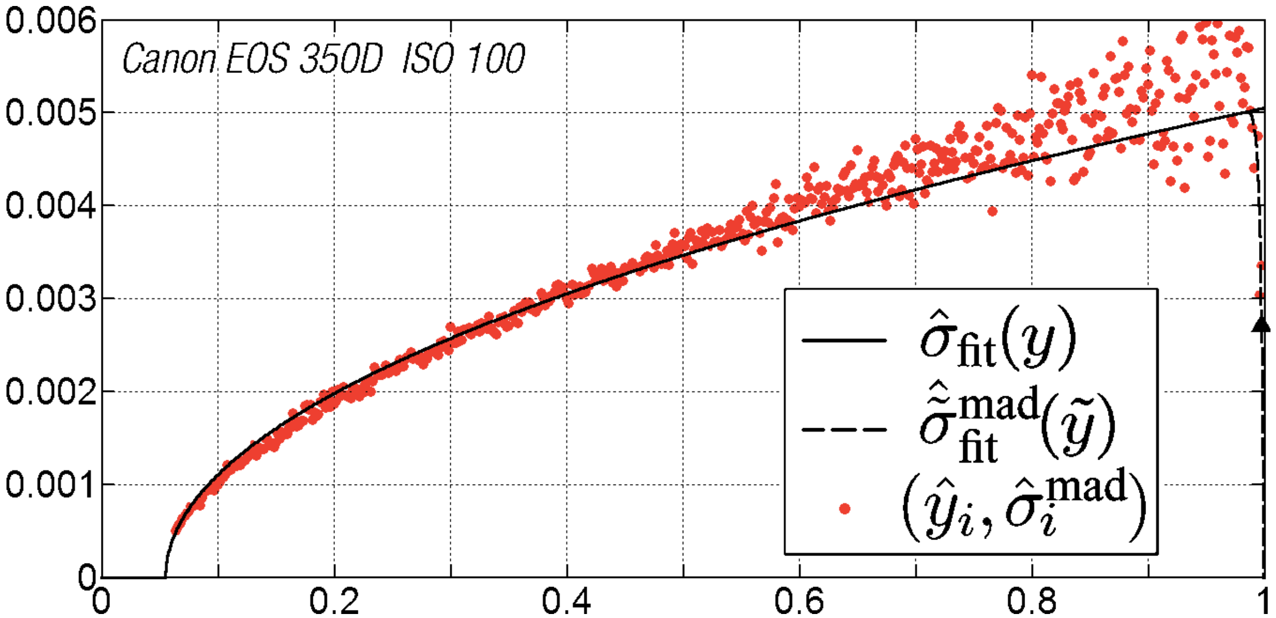


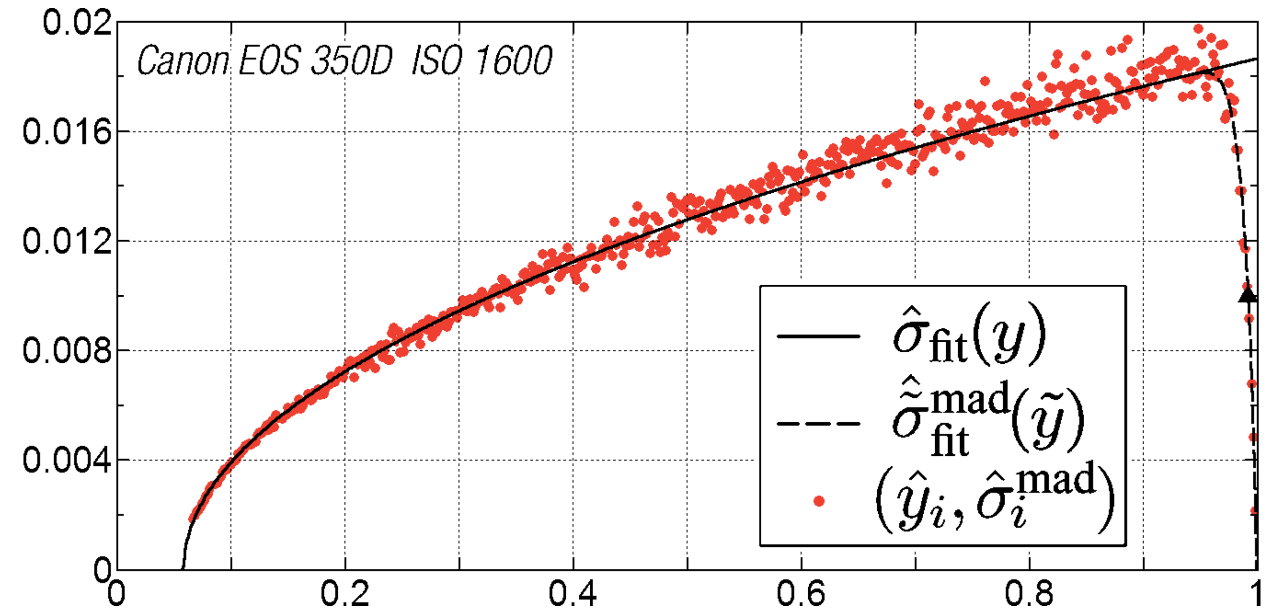
[Introduction to Color Processing in Digital Cameras, Süsstrunk]

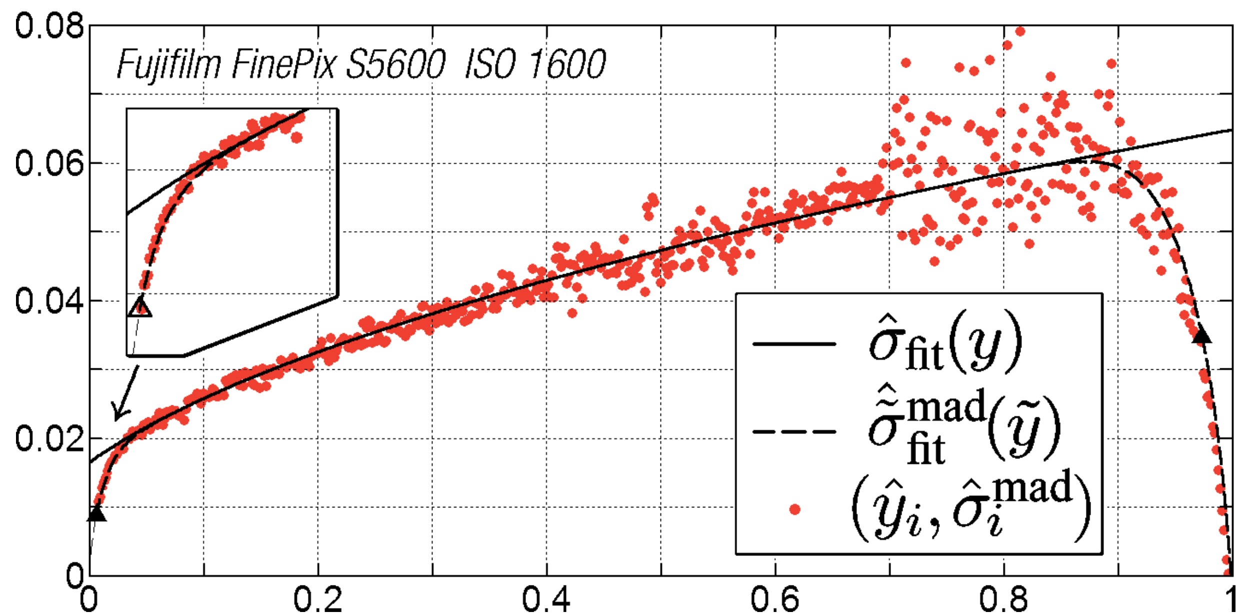
Noise model

[Foi *et al.*, IEEE TIP]

$$v[\mathbf{k}] = \mathcal{C}(\mathrm{r}^X[\mathbf{k}] + \varepsilon[\mathbf{k}]) \qquad \varepsilon[\mathbf{k}] \sim \sigma[\mathbf{k}]\,\mathcal{N}(0,1) \qquad \sigma[\mathbf{k}] = \sqrt{a\,\mathrm{r}^X[\mathbf{k}] + b}$$







Simplified acquisition model

- We ignore:
  - the spectral sensitivity functions of the R,G,B filters
  - cross-talk
  - non-linearities of the sensor, A/D conversion
  - white balancing
  - optical blur due to the optical system

☞

$$v[\mathbf{k}] = \mathcal{C}\left(\mathcal{G}^{-1}(\mathrm{im}^X[\mathbf{k}]) + \sqrt{a\,\mathcal{G}^{-1}(\mathrm{im}^X[\mathbf{k}]) + b}\,e[\mathbf{k}]\right)$$

clipping

inverse of tone mapping

$$\mathcal{G}(x)^{-1} \approx x^{2.2}$$

Reconstruction procedure

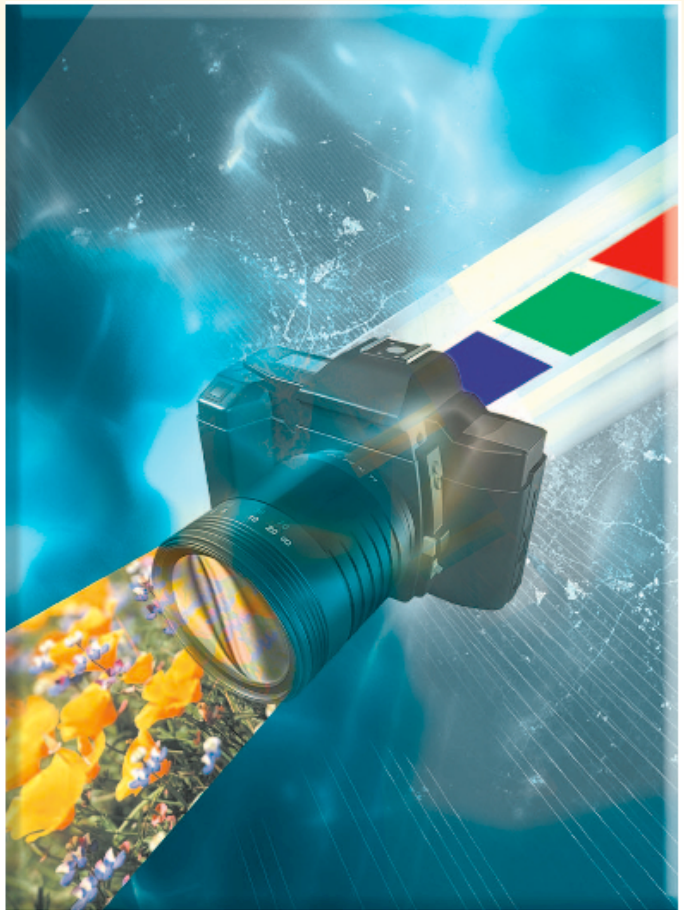
$$v[\mathbf{k}] = \mathcal{C}\left(\mathcal{G}^{-1}(\mathrm{im}^X[\mathbf{k}]) + \sqrt{a\,\mathcal{G}^{-1}(\mathrm{im}^X[\mathbf{k}]) + b}\,e[\mathbf{k}]\right)$$

1) variance stabilization (clipping taken into account)

2) joint demosaicking/denoising in the AWGN setting

3) pixel-wise mapping: bias correction $E\{f(x)\} \neq f(E\{x\})$
+ inverse stabilization + unclipping + tone mapping

Outline



- The challenge of demosaicking
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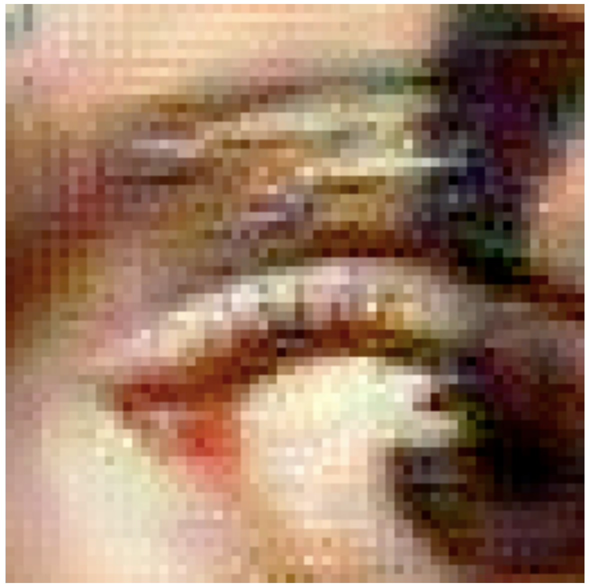
Naive approaches to joint demosaicking/denoising

Original image

Demosaicked image

Demosaicking + denoising



Denoising + demosaicking

Joint demosaicking/ denoising [Hirakawa, 2006]

Ad hoc approaches

- Hirakawa *et al.* “Joint demosaicing and denoising”, *IEEE TIP*, 2006

Linear demosaicking: behavior under noise

$$v[\mathbf{k}] = v_0[\mathbf{k}] + \varepsilon[\mathbf{k}] \qquad\qquad \varepsilon[\mathbf{k}] \sim \mathcal{N}(0, \sigma^2)$$

- Let d_0 be the demosaicked image in absence of noise

$$\mathbf{d}[\mathbf{k}] = \mathbf{d}_0[\mathbf{k}] + \mathbf{e}[\mathbf{k}]$$

- The demosaicked color noise $\mathbf{e}$ is such that:
  - $e^{G/M}$, $e^{R/B}$, e^L are independent Gaussian noise realizations
  - $e^{G/M}$ is stationary with spectral density $\frac{8}{3}\sigma^2|\hat{h}_{G/M}(\boldsymbol{\omega})|^2$
  - $e^{R/B}$ is stationary with spect. dens. $2\sigma^2\left(|\hat{h}_{R/B}(\omega_1,\omega_2)|^2 + |\hat{h}_{R/B}(\omega_2,\omega_2)|^2\right)$
  - e^L is not stationary and not white

→ The basis $\mathbf{L}, \mathbf{C}^{G/M}, \mathbf{C}^{R/B}$ is appropriate to address the problem

MMSE chrominance filters

→ The chrominance should be denoised before estimating the luminance

- Wiener-like FIR chrominance filters of size $N \times N$ optimal for a learning image base: linear systems of size $N^2 \times N^2$ to solve:

$$\mathbf{A}_{G/M}\mathbf{h}_{G/M} = \mathbf{b}_{G/M}$$
$$\mathbf{A}_{R/B}\mathbf{h}_{R/B} = \mathbf{b}_{R/B}$$

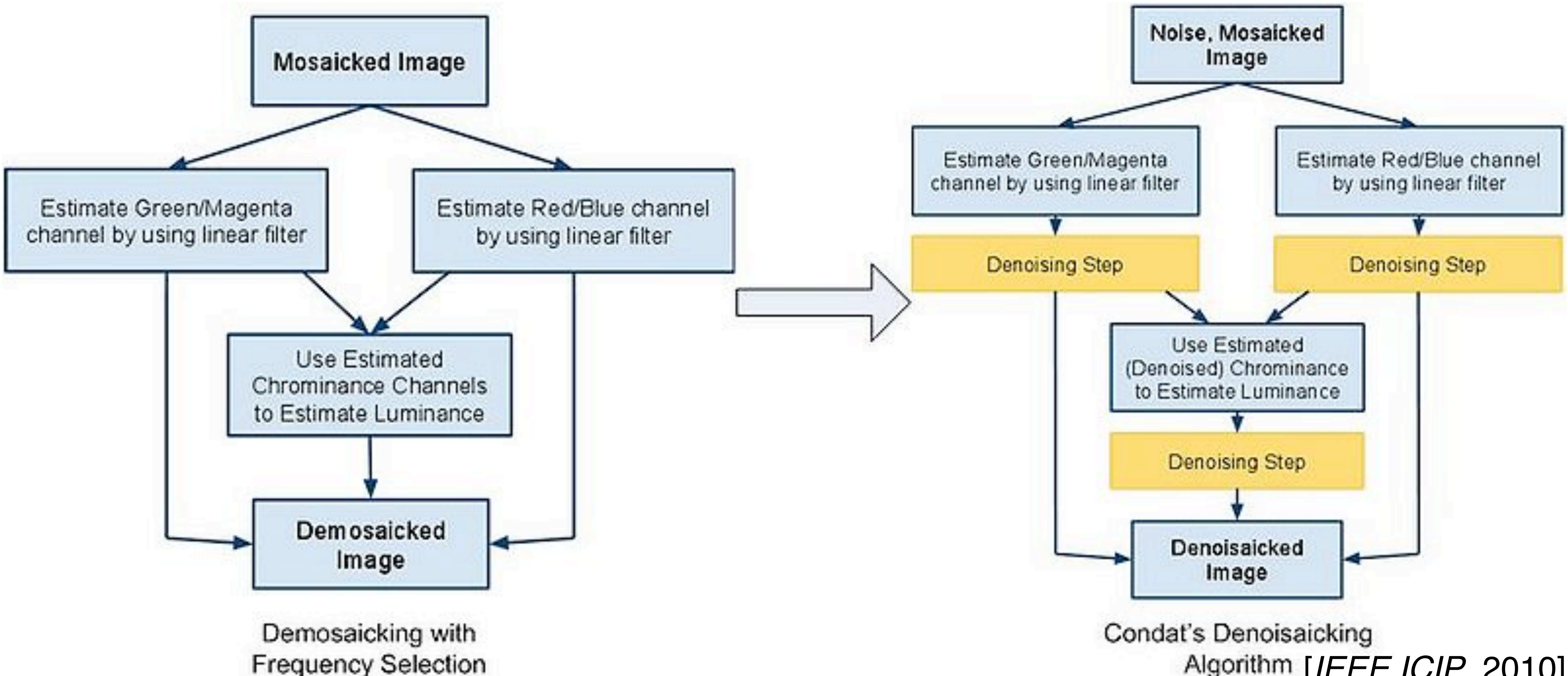
[Dubois, *IEEE ICIP*, 2006]

- In presence of noise:

$$(\mathbf{A}_{G/M} + \tfrac{8}{3}\sigma^2\mathbf{I})\mathbf{h}_{G/M} = \mathbf{b}_{G/M}$$
$$(\mathbf{A}_{R/B} + 4\sigma^2\mathbf{I})\mathbf{h}_{R/B} = \mathbf{b}_{R/B}$$

[Condat, *IEEE ICIP*, 2010]

Strategy by frequency selection + luminance denoising



[*IEEE ICIP*, 2010]

$$v[\mathbf{k}] = \frac{1}{\sqrt{3}}u^L[\mathbf{k}] + \frac{1}{\sqrt{24}}u^{G/M}[\mathbf{k}] + \frac{\sqrt{6}}{4}(-1)^{k_1+k_2}u^{G/M}[\mathbf{k}] + \frac{\sqrt{2}}{4}(-1)^{k_2}u^{R/B}[\mathbf{k}] - \frac{\sqrt{2}}{4}(-1)^{k_1}u^{R/B}[\mathbf{k}] + \varepsilon[\mathbf{k}]$$

Results $\sigma = 20$

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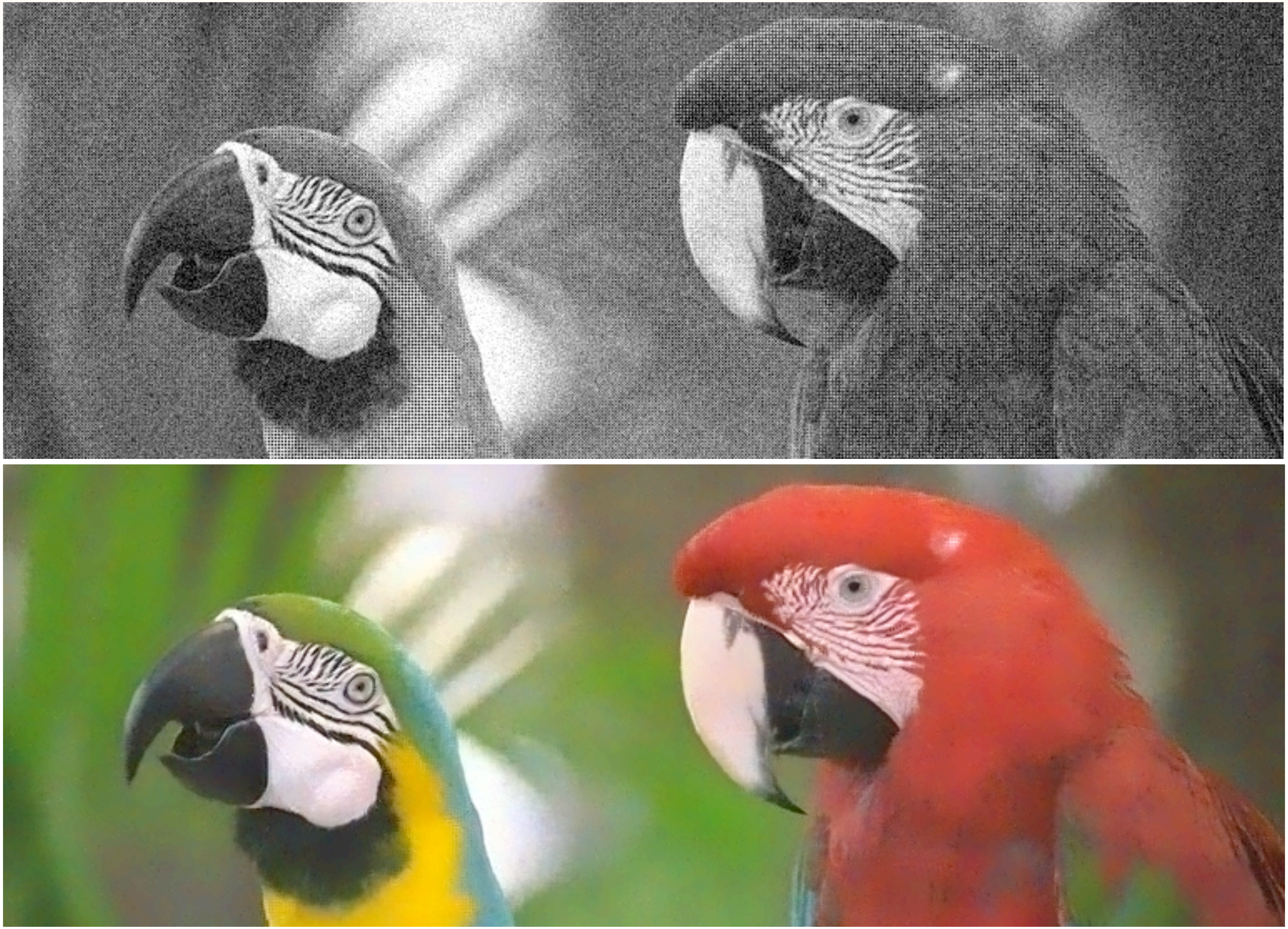


Results

Original Image

gipsa-lab

$\sigma = 20$

Results

Hirakawa *et al.*, *IEEE TIP,* 2006

$\sigma = 20$

Results

Zhang *et al., IEEE TIP,* 2007

$\sigma = 20$

Results

Zhang *et al.*, *IEEE TIP*, 2009

$\sigma = 20$

Results

Paliy *et al., Int. J. Im. Sys. and Tech.*, 2007

$\sigma = 20$

Results

Proposed

A variational interpretation

[Condat, *GRETSI*, 2009]

- We can show that demosaicking by frequency selection solves the following variational problem:

$$\text{minimize}_{\mathbf{d}} \quad \mu\|\nabla d^L\|_{\ell_2}^2 + \|\nabla d^{G/M}\|_{\ell_2}^2 + \|\nabla d^{R/B}\|_{\ell_2}^2 \qquad s.t. \quad d^{X[\mathbf{k}]} = v[\mathbf{k}],\ \forall \mathbf{k}$$

- Key point: the chrominance energy is more penalized: $\mu \approx 0.05$
- Remark 1: the solution does not depend on the choice of the chrominance basis.
- Remark 2: this generic approach can be used with every CFA.

Improvement: minimize the TV

- New denoisaicking strategy:
  - step 1) solve

$$\text{minimize}_{\mathbf{d}} \quad \|\mathbf{d}\|_{\text{TV}} := \mu \left\| \sqrt{(\nabla_x d^L)^2 + (\nabla_y d^L)^2} \right\|_{\ell_1} +$$

$$\left\| \sqrt{(\nabla_x d^{G/M})^2 + (\nabla_x d^{R/B})^2 + (\nabla_y d^{G/M})^2 + (\nabla_y d^{R/B})^2} \right\|_{\ell_1}$$

$$s.t. \quad d^{X[\mathbf{k}]} = v[\mathbf{k}], \ \forall \mathbf{k}$$

  - step 2) denoise d^L

Primal-dual optimization algorithm

[Chambolle and Pock, 2011, "A first-order primal-dual algorithm for convex problems with applications to imaging"]

- Choose $\alpha > 0$, set $\beta = 1/(8\alpha)$, $\mathbf{b} = (0)$
- Iterate
  - $\forall X \in \{R,G,B\},\ \mathbf{b}^X_{(n+1)} = \mathbf{b}^X_{(n)} + \alpha \nabla \bar{d}^X_{(n)}$
  - $\forall \mathbf{k} \in \mathbb{Z}^2,\ \mathbf{b}^L_{(n+1)}[\mathbf{k}] = \dfrac{\mathbf{b}^L_{(n+1)}[\mathbf{k}]}{\max(1, |\mathbf{b}^L_{(n+1)}[\mathbf{k}]|/\mu)}$
  - $\forall \mathbf{k} \in \mathbb{Z}^2,\ \forall X \in \{G/M, R/B\},\ \mathbf{b}^X_{(n+1)}[\mathbf{k}] = \dfrac{\mathbf{b}^X_{(n+1)}[\mathbf{k}]}{\max(1, \sqrt{|\mathbf{b}^{G/M}_{(n+1)}[\mathbf{k}]|^2 + |\mathbf{b}^{R/B}_{(n+1)}[\mathbf{k}]|^2})}$
  - $\forall X \in \{R,G,B\},\ d^X_{(n+1)} = d^X_{(n)} + \beta \,\mathrm{div}\, \mathbf{b}^X_{(n+1)}$
  - $\forall \mathbf{k} \in \mathbb{Z}^2,\ d^{X[\mathbf{k}]}_{(n+1)}[\mathbf{k}] = v[\mathbf{k}]$
  - $\bar{\mathbf{d}}_{(n+1)} = 2\mathbf{d}_{(n+1)} - \mathbf{d}_{(n)}$

$\sigma = 20$

Result

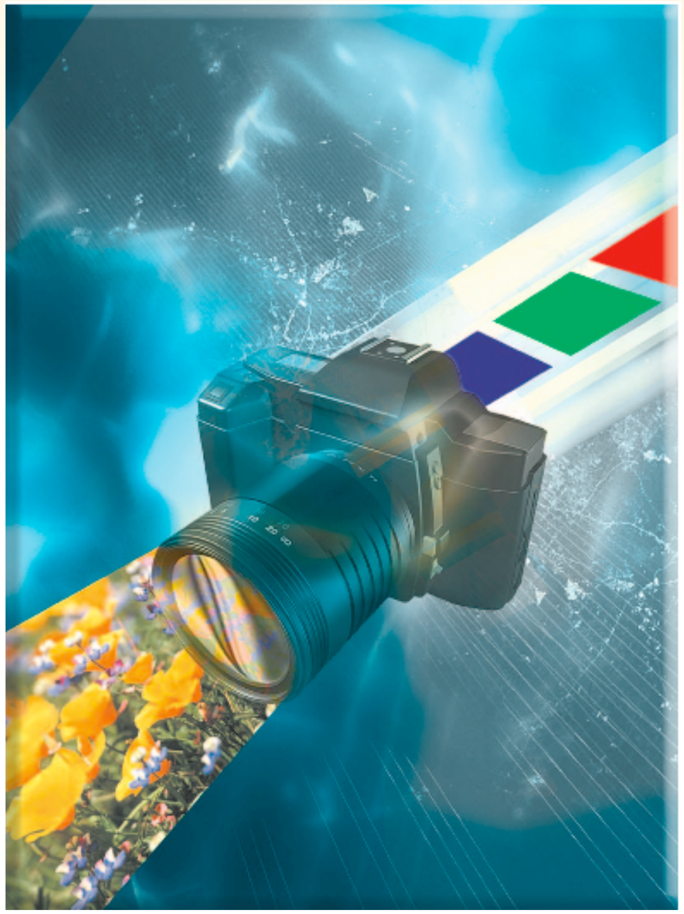
Method by frequency selection

$\sigma = 20$

Result

Method by TV minimization

Outline



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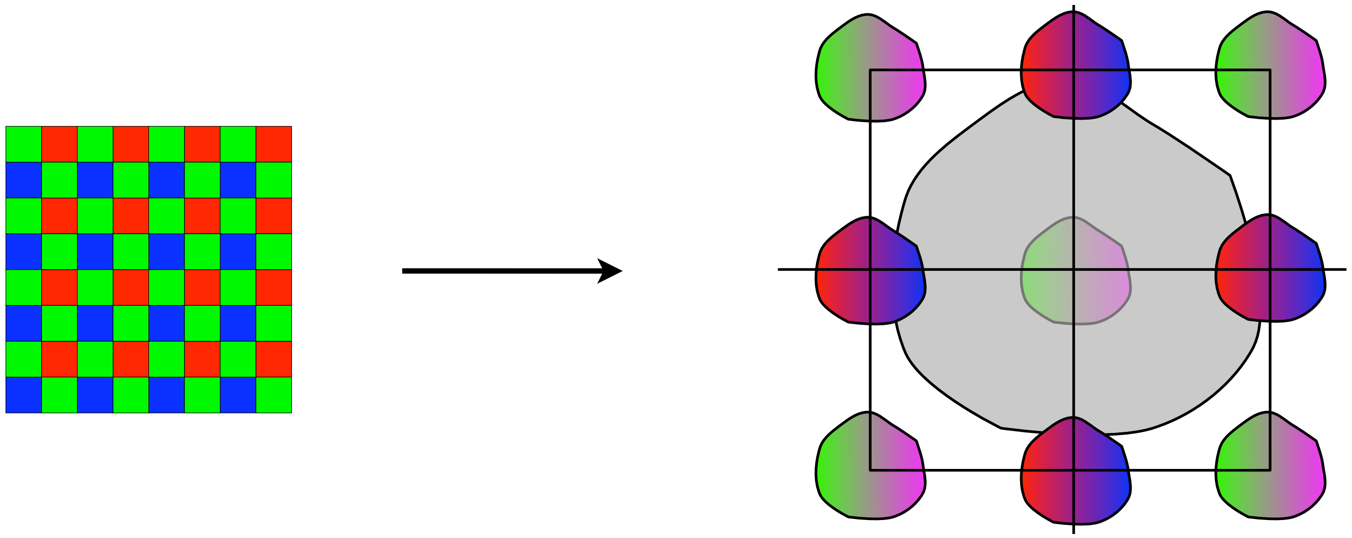
Choice of the CFA: a packing problem

- Fourier interpretation of mosaicking:

$$\hat{v}(\boldsymbol{\omega}) = \sum_{X \in \{L, C_1, C_2\}} \widehat{u^X}(\boldsymbol{\omega}) * \widehat{\mathrm{cfa}^X}(\boldsymbol{\omega}), \quad \boldsymbol{\omega} \in \mathbb{R}^2$$



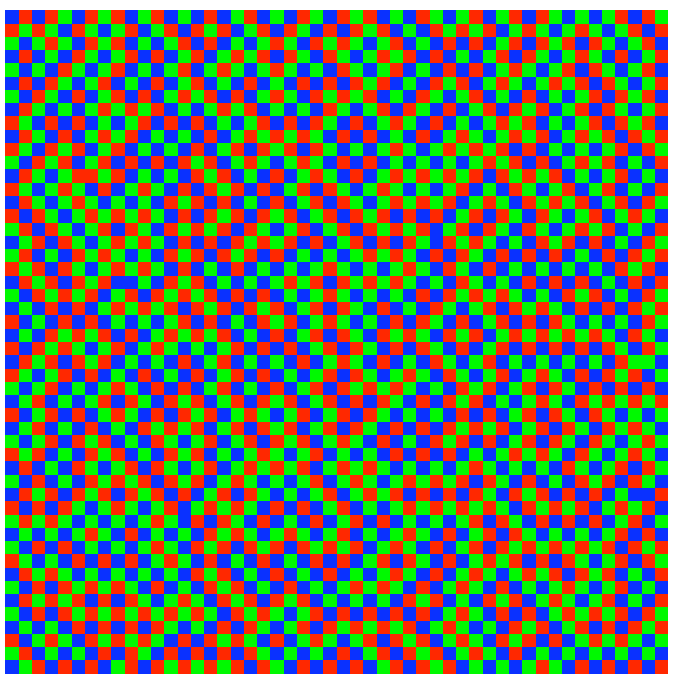
→ Idea of Hirakawa [IEEE TIP, 2008] to design the CFA directly in Fourier domain

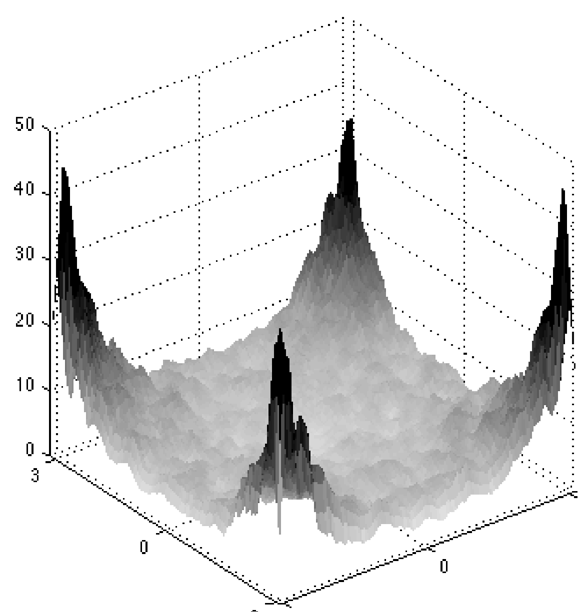
- luminance in the baseband
- chrominance far away from the luminance

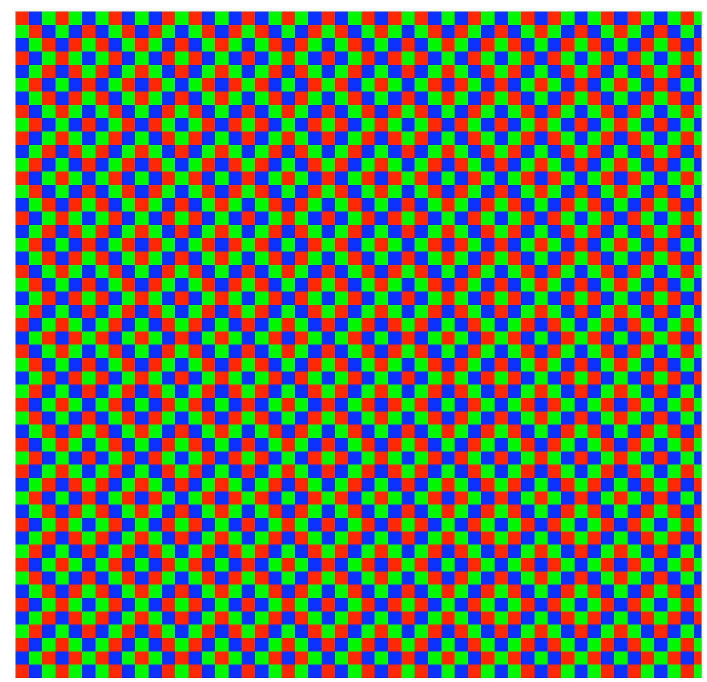
Choice of a R,G,B CFA

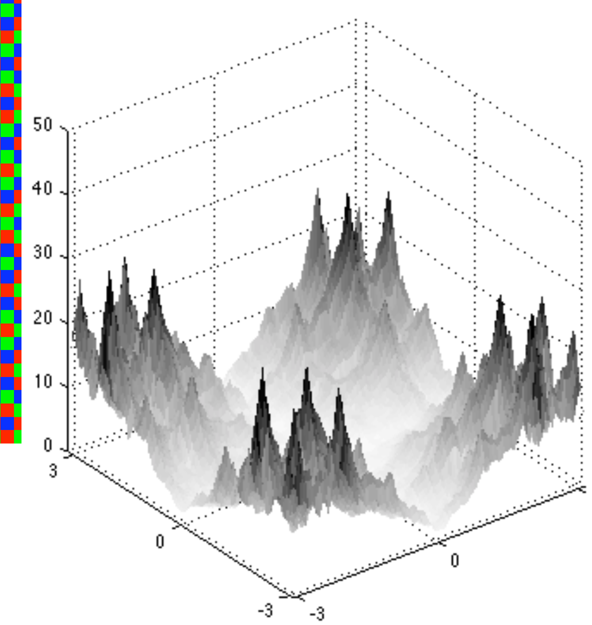
- Periodic patterns: the Bayer CFA is optimal
- → can we do better with aperiodic CFAs? [Condat, IVC, IEEE ICIP]

$$\hat{v}(\boldsymbol{\omega}) = \frac{1}{\sqrt{3}}\widehat{u^L}(\boldsymbol{\omega}) + \sum_{C\in\{C_1,C_2\}} \widehat{u^C}(\boldsymbol{\omega}) * \widehat{\mathrm{cfa}^C}(\boldsymbol{\omega}), \quad \boldsymbol{\omega} \in \mathbb{R}^2$$









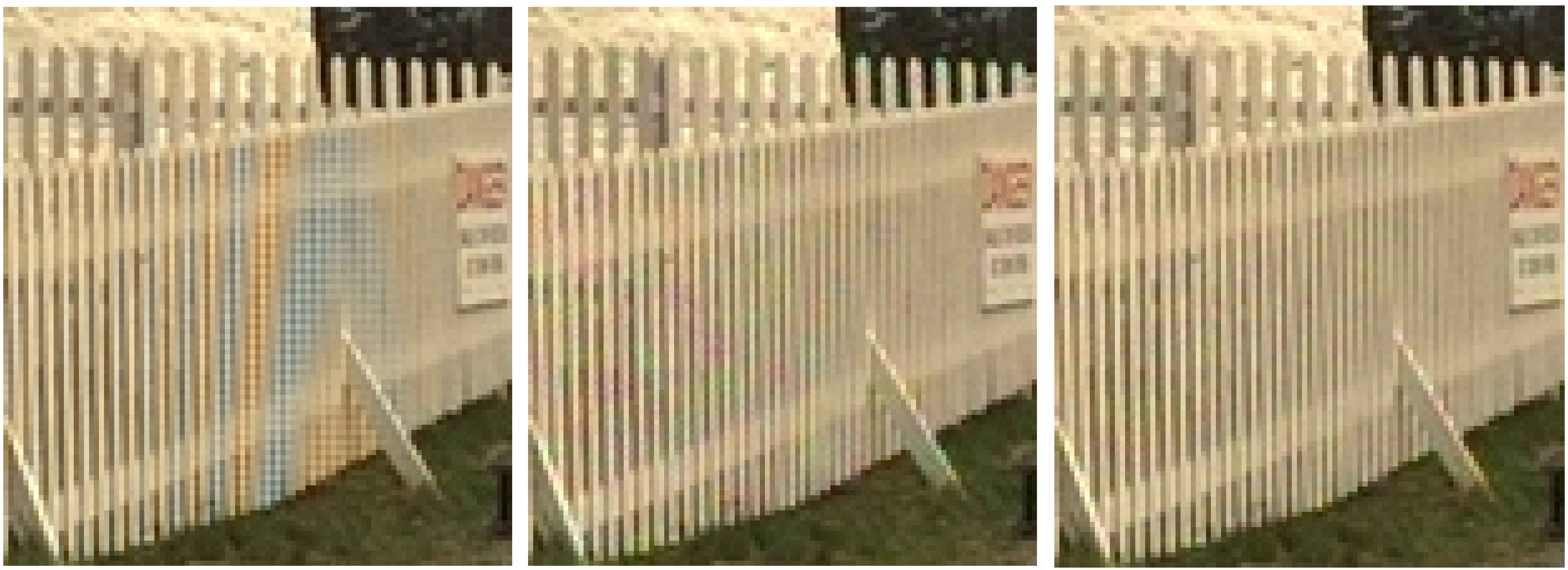
Blue noise spectral characteristics (used in halftoning...)

Generic variational demosaicking

$$\text{minimize}_{\mathbf{d}} \quad \mu\|\nabla d^L\|_{\ell_2}^2 + \|\nabla d^{G/M}\|_{\ell_2}^2 + \|\nabla d^{R/B}\|_{\ell_2}^2 \qquad s.t. \quad d^{X[\mathbf{k}]} = v[\mathbf{k}],\ \forall \mathbf{k}$$

- Quadratic problem → linear system to solve
- Iterative method (Jacobi)

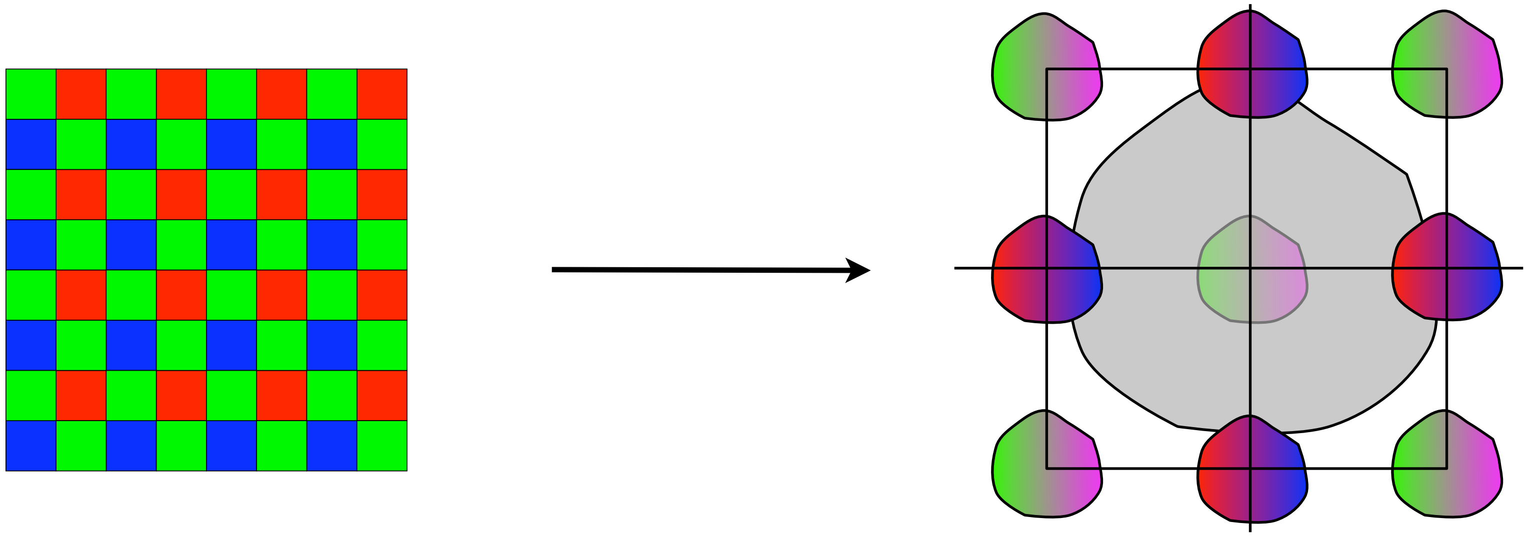
[Condat, IEEE ICIP, 2009]



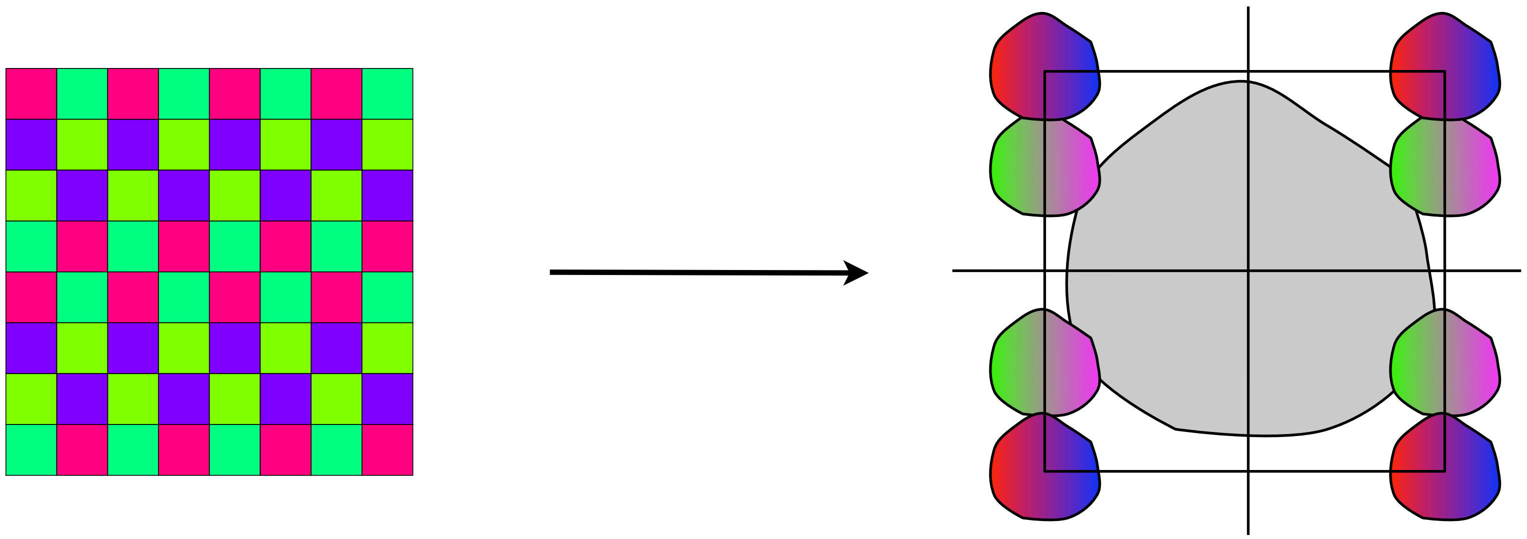
Bayer — random CFA, type I — random CFA, type II

Periodic CFAs with arbitrary colors

[Bayer]

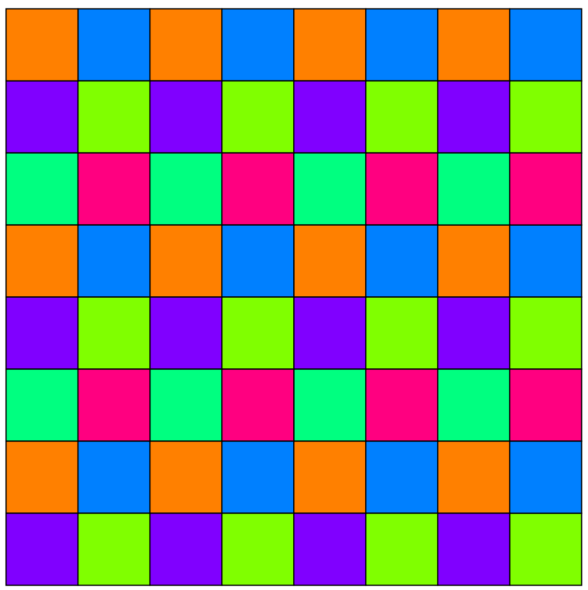


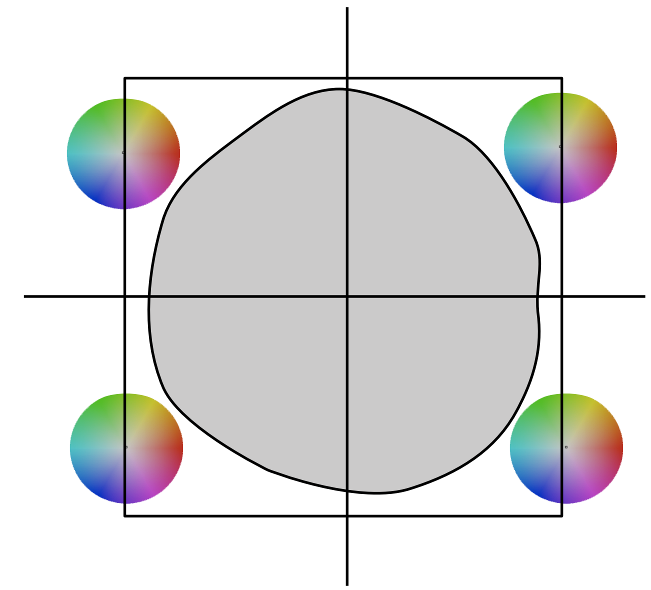
[Hirakawa,
IEEE ICIP 2008]



Proposed 2x3 CFA with 6 colors

[Condat, IEEE TIP 2011]





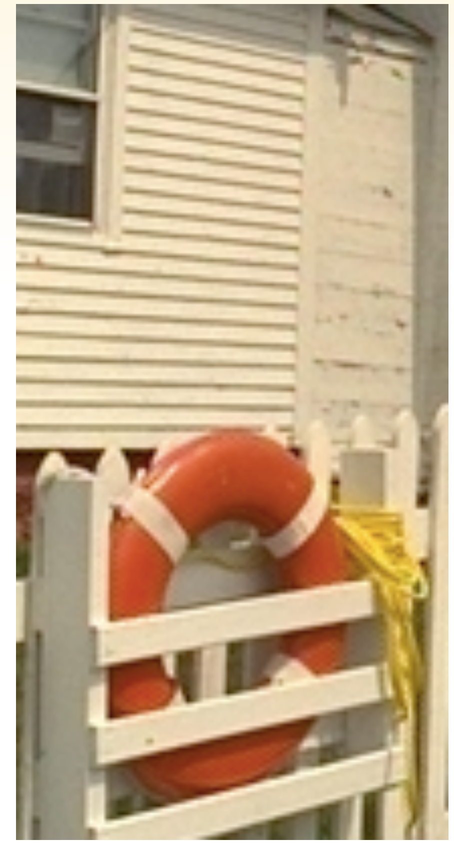
- 6 couleurs [1,½,0], [0,½,1], [½,0,1], [½,1,0], [0,1,½], [1,0,½]
- Color isotropy: two chrominance channels modulated in quadrature at the same frequency
- Designed to maximize $\gamma^C, \ \gamma^L$

$\rightarrow \quad \omega_0 = \dfrac{2\pi}{3}$

$$\mathrm{cfa}^L[\mathbf{k}] = \gamma^L,$$
$$\mathrm{cfa}^{R/B}[\mathbf{k}] = \gamma^C(-1)^{k_1}\sqrt{2}\sin(\omega_0 k_2 - \varphi)$$
$$\mathrm{cfa}^{V/M}[\mathbf{k}] = \gamma^C(-1)^{k_1}\sqrt{2}\cos(\omega_0 k_2 - \varphi)$$

Results

Bayer

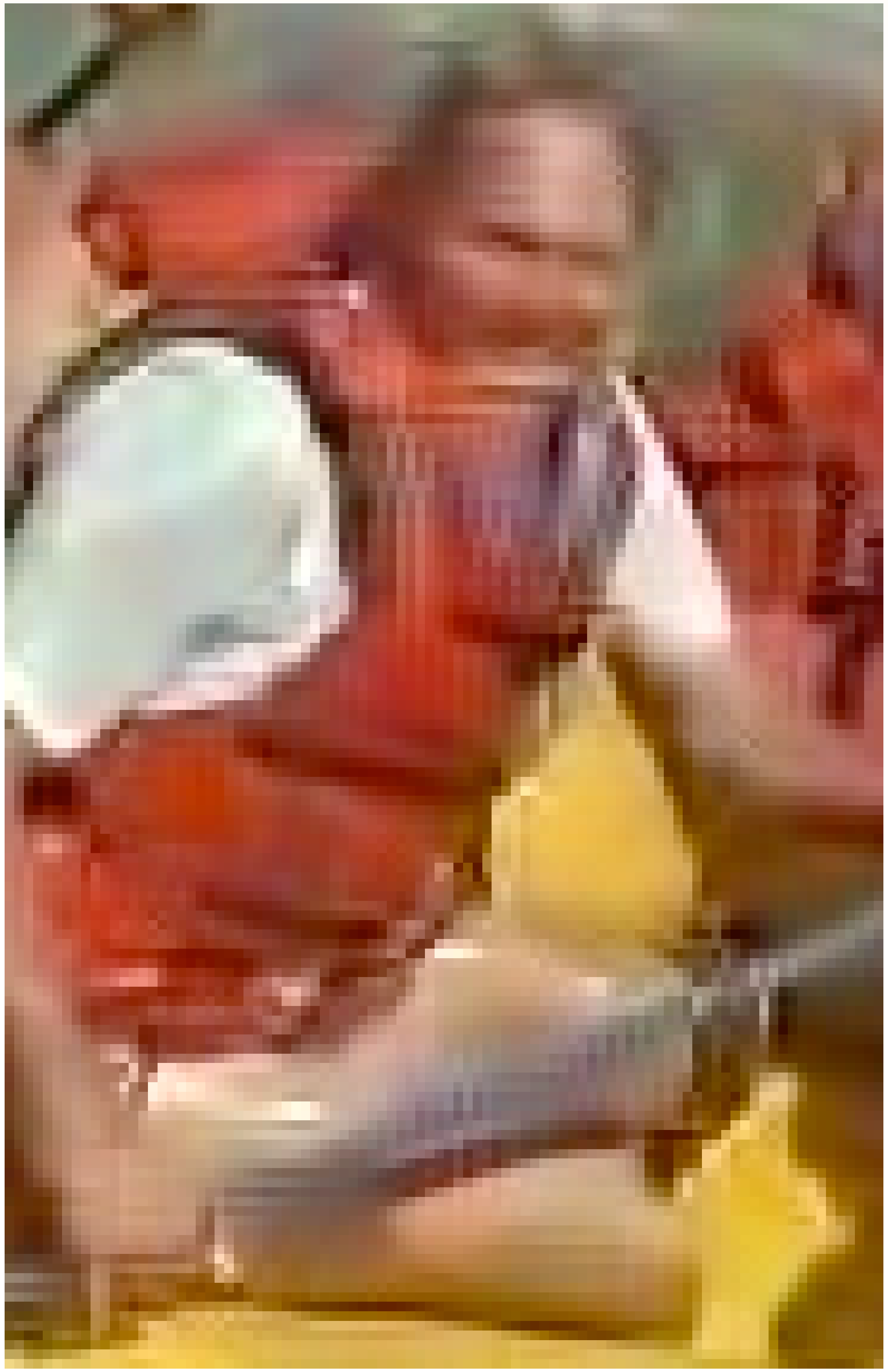


Condat

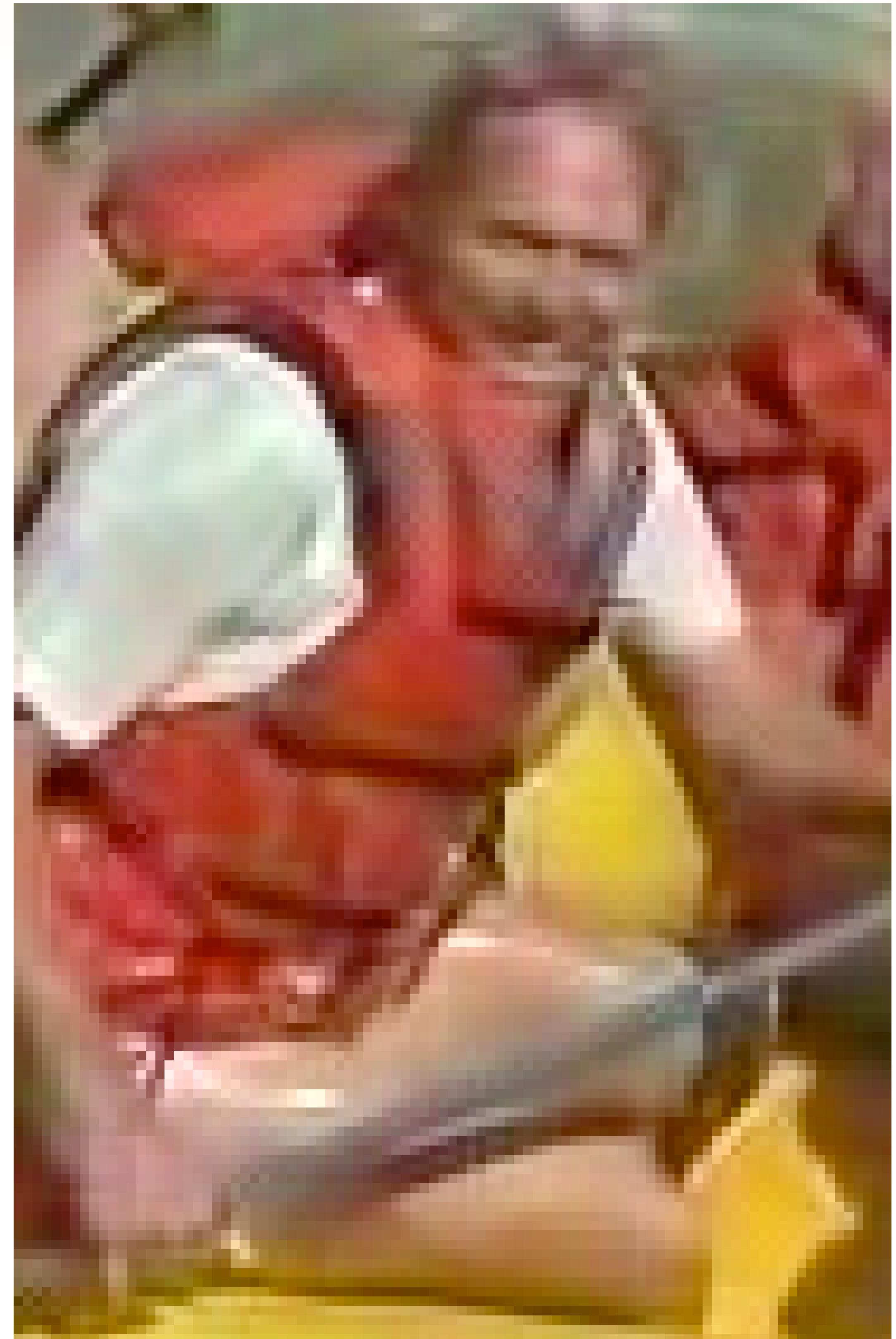
Results

$\sigma = 40$

Bayer



Condat



Conclusion

- Demosaicking by frequency selection
  - linear (simple, stable), fast, efficient
  - noisy case: just demosaick and denoise the luminance
  - variational interpretation: generic, extension to non-quadratic penalty
- Can be inserted in a realistic reconstruction pipeline using variance stabilization
- There are better alternatives to the Bayer CFA (more robust to aliasing and noise)
  - R,G,B aperiodic CFAs
  - 2x3 periodic CFA