Models and methods for the acquisition of color images

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A Bayer color filter array (CFA) is overlaid on the sensor.

What you see
Color image acquisition with a single sensor

A (Bayer) color filter array (CFA) is overlaid on the sensor.

what your camera sees
Outline

• The challenge of demosaicking
• An image formation model
• «Denoisaicking» methods
• New robust CFAs
Luminance / chrominance basis

\[
        \begin{align*}
        L &= \frac{1}{\sqrt{3}} [1, 1, 1]^T \\
        C_{G/M} &= \frac{1}{\sqrt{6}} [-1, 2, -1]^T \\
        C_{R/B} &= \frac{1}{\sqrt{3}} [1, 0, -1]^T 
        \end{align*}
\]
Chrominance of natural images

Blue / red

Chrominance of all the pixels in a base of 150 images

Magenta / green
Frequency interpretation of Bayer sampling

\[ \hat{v}(\omega) = \frac{1}{\sqrt{3}} \hat{u}^L(\omega) + \frac{1}{\sqrt{24}} \hat{u}^{G/M}(\omega) + \frac{\sqrt{6}}{4} \hat{u}^{G/M}(\omega - [\pi, \pi]^T) + \frac{\sqrt{2}}{4} \hat{u}^{R/B}(\omega - [0, \pi]^T) - \frac{\sqrt{2}}{4} \hat{u}^{R/B}(\omega - [\pi, 0]^T) \]
Frequency interpretation of Bayer sampling

\[ v[k] = \frac{1}{\sqrt{3}} u^L[k] + \frac{1}{\sqrt{24}} u^{G/M}[k] + \frac{\sqrt{6}}{4} (-1)^{k_1+k_2} u^{G/M}[k] + \frac{\sqrt{2}}{4} (-1)^{k_2} u^{R/B}[k] - \frac{\sqrt{2}}{4} (-1)^{k_1} u^{R/B}[k] \]
Linear demosaicking by frequency selection

- Chrominance obtained by modulation + lowpass filtering
  
  \[ d^{G/M} = \frac{4}{\sqrt{6}} v_{\pi,\pi} \ast h_{G/M} \text{ where } v_{\pi,\pi}[k] = (-1)^{k_1+k_2} v[k] \]
  
  \[ d^{R/B}_H = -2\sqrt{2} v_{\pi,0} \ast h_{R/B} \text{ where } v_{\pi,0}[k] = (-1)^{k_1} v[k] \]
  
  \[ d^{R/B}_V = 2\sqrt{2} v_{0,\pi} \ast (h_{R/B})^T \text{ where } v_{0,\pi}[k] = (-1)^{k_2} v[k] \]
  
  \[ d^{R/B} = \frac{1}{2} (d^{R/B}_H + d^{R/B}_V) \]

- Luminance as the residual
  
  \[ \frac{1}{\sqrt{3}} d^L = v[k] - \left( \frac{1}{\sqrt{24}} + \frac{\sqrt{6}}{4} (-1)^{k_1+k_2} \right) d^{G/M}[k] - \frac{\sqrt{2}}{4} \left( (-1)^{k_2} - (-1)^{k_1} \right) d^{R/B}[k] \]

[Dubois, IEEE SPL, 2005]
Result

- Aliasing artifacts due to high-frequency content of luminance
Redundancy of the blue/red chrominance information

locally:
Adaptive demosaicking based on the structure tensor
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The workflow of digital photography

[Image Representation in \textit{raw sensor} coordinates (device dependent)]

[Camera Compensation: Exposure Adjustments, White Balancing, Sensor Characterization, Color Transform]

[Image Representation in \textit{input-referred encoding} (device independent)]

[Scene Editing]

[Color Rendering: Tone Mapping, Gamut Mapping, Color Preference (Intent)]

[Image Representation in \textit{output-referred encoding} (device independent)]

[Picture Editing]

[Introduction to Color Processing in Digital Cameras, Süsstrunk]
Noise model

\[ v[k] = C(r^X[k] + \varepsilon[k]) \]

\[ \varepsilon[k] \sim \sigma[k] \mathcal{N}(0, 1) \]

\[ \sigma[k] = \sqrt{a \cdot r^X[k] + b} \]

[Foi et al., IEEE TIP]
Simplified acquisition model

- We ignore:
  - the spectral sensitivity functions of the R,G,B filters
  - cross-talk
  - non-linearities of the sensor, A/D conversion
  - white balancing
  - optical blur due to the optical system

\[ v[k] = C \left( \mathcal{G}^{-1}(im^X[k]) + \sqrt{a \mathcal{G}^{-1}(im^X[k]) + b e[k]} \right) \]

- clipping
- inverse of tone mapping
  \[ \mathcal{G}(x)^{-1} \approx x^{2.2} \]
Reconstruction procedure

\[ v[k] = C \left( \mathcal{G}^{-1}(\text{im}^X[k]) + \sqrt{a \mathcal{G}^{-1}(\text{im}^X[k]) + b e[k]} \right) \]

1) variance stabilization (clipping taken into account)
2) joint demosaicking/denoising in the AWGN setting
3) pixel-wise mapping: bias correction \( E\{f(x)\} \neq f(E\{x\}) \)
   + inverse stabilization + unclipping + tone mapping
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Naive approaches to joint demosaicking/denoising

Original image
Demosaicked image

Demosaicking + denoising
Denoising + demosaicking
Joint demosaicking/denoising [Hirakawa, 2006]
Ad hoc approaches

Let $d_0$ be the demosaicked image in absence of noise.

\[ d[k] = d_0[k] + e[k] \]

The demosaicked color noise $e$ is such that:

- $e^{G/M}, e^{R/B}, e^L$ are independent Gaussian noise realizations.
- $e^{G/M}$ is stationary with spectral density \( \frac{8}{3} \sigma^2 |\hat{h}_{G/M}(\omega)|^2 \).
- $e^{R/B}$ is stationary with spect. dens. \( 2\sigma^2 (|\hat{h}_{R/B}(\omega_1, \omega_2)|^2 + |\hat{h}_{R/B}(\omega_2, \omega_2)|^2) \).
- $e^L$ is not stationary and not white.

\[ \rightarrow \text{The basis } L, C^{G/M}, C^{R/B} \text{ is appropriate to address the problem} \]
MMSE chrominance filters

The chrominance should be denoised before estimating the luminance.

- Wiener-like FIR chrominance filters of size $N \times N$ optimal for a learning image base: linear systems of size $N^2 \times N^2$ to solve:

\[
A_{G/M} h_{G/M} = b_{G/M} \\
A_{R/B} h_{R/B} = b_{R/B}
\]

- In presence of noise:

\[
(A_{G/M} + \frac{8}{3} \sigma^2 I) h_{G/M} = b_{G/M} \\
(A_{R/B} + 4\sigma^2 I) h_{R/B} = b_{R/B}
\]

[Dubois, IEEE ICIP, 2006]

[Condat, IEEE ICIP, 2010]
Strategy by frequency selection + luminance denoising

\[
v[k] = \frac{1}{\sqrt{3}} u^L[k] + \frac{1}{\sqrt{24}} u^{G/M}[k] + \frac{\sqrt{6}}{4} (-1)^{k_1 + k_2} u^{G/M}[k] + \\
\frac{\sqrt{2}}{4} (-1)^{k_2} u^{R/B}[k] - \frac{\sqrt{2}}{4} (-1)^{k_1} u^{R/B}[k] + \varepsilon[k]
\]
Results

$\sigma = 20$
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Results  \[ \sigma = 20 \]
Results

Original Image
σ = 20

Results

Hirakawa et al., IEEE TIP, 2006
Results

Zhang et al., IEEE TIP, 2007
Results

Zhang et al., IEEE TIP, 2009

\[ \sigma = 20 \]
Results

\(\sigma = 20\)

Paliy et al., *Int. J. Im. Sys. and Tech.*, 2007
Results

\[ \sigma = 20 \]

Proposed
A variational interpretation

[Condat, GRETSI, 2009]

- We can show that demosaicking by frequency selection solves the following variational problem:

\[
\text{minimize } \mu \| \nabla d^L \|_{\ell_2}^2 + \| \nabla d^{G/M} \|_{\ell_2}^2 + \| \nabla d^{R/B} \|_{\ell_2}^2 \quad \text{s.t. } d^X[k] = v[k], \quad \forall k
\]

- Key point: the chrominance energy is more penalized: \( \mu \approx 0.05 \)

- Remark 1: the solution does not depend on the choice of the chrominance basis.

- Remark 2: this generic approach can be used with every CFA.
Improvement: minimize the TV

- New denoising strategy:
  
  - step 1) solve
    
    \[
    \text{minimize } d \quad \|d\|_{TV} := \mu \left\| \sqrt{(\nabla_x d^L)^2 + (\nabla_y d^L)^2} \right\|_{\ell_1} + \\
    \left\| \sqrt{(\nabla_x d^{G/M})^2 + (\nabla_x d^{R/B})^2 + (\nabla_y d^{G/M})^2 + (\nabla_y d^{R/B})^2} \right\|_{\ell_1}
    \]
    
    \[s.t. \quad d^X[k] = v[k], \quad \forall k\]

  - step 2) denoise \(d^L\)
Primal-dual optimization algorithm

[Chambolle and Pock, 2011, “A first-order primal-dual algorithm for convex problems with applications to imaging”]

- **Choose** \( \alpha > 0 \), **set** \( \beta = 1/(8\alpha) \), \( b = (0) \)

- **Iterate**
  - \( \forall X \in \{R, G, B\}, \ b^X_{(n+1)} = b^X_{(n)} + \alpha \nabla \bar{d}^X_{(n)} \)
  - \( \forall k \in \mathbb{Z}^2, \ b^L_{(n+1)}[k] = \frac{b^L_{(n+1)}[k]}{\max(1, |b^L_{(n+1)}[k]|/\mu)} \)
  - \( \forall k \in \mathbb{Z}^2, \forall X \in \{G/M, R/B\}, \ b^X_{(n+1)}[k] = \frac{b^X_{(n+1)}[k]}{\max(1, \sqrt{|b^{G/M}_{(n+1)}[k]|^2 + |b^{R/B}_{(n+1)}[k]|^2})} \)
  - \( \forall X \in \{R, G, B\}, \ d^X_{(n+1)} = d^X_{(n)} + \beta \text{div} b^X_{(n+1)} \)
  - \( \forall k \in \mathbb{Z}^2, \ d^X[k]_{(n+1)} = v[k] \)
  - \( \bar{d}_{(n+1)} = 2d_{(n+1)} - d_{(n)} \)
Result

Method by frequency selection

$\sigma = 20$
Result

Method by TV minimization

$\sigma = 20$
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Choice of the CFA: a packing problem

- Fourier interpretation of mosaicking:
  \[ \hat{v}(\omega) = \sum_{X \in \{L, C_1, C_2\}} \hat{u}^X(\omega) \ast \hat{\text{cfa}}^X(\omega), \quad \omega \in \mathbb{R}^2 \]

→ Idea of Hirakawa [IEEE TIP, 2008] to design the CFA directly in Fourier domain

- luminance in the baseband
- chrominance far away from the luminance
Choice of a R,G,B CFA

- Periodic patterns: the Bayer CFA is optimal
- → can we do better with aperiodic CFAs?

\[ \hat{v}(\omega) = \frac{1}{\sqrt{3}} \hat{u}^L(\omega) + \sum_{C \in \{C_1, C_2\}} \hat{u}^C(\omega) \ast \text{cfa}^C(\omega), \quad \omega \in \mathbb{R}^2 \]

Blue noise spectral characteristics (used in halftoning...)

[Condat, IVC, IEEE ICIP]
Generic variational demosaicking

\[
\text{minimize } \mu \| \nabla d^L \|_2^2 + \| \nabla d^{G/M} \|_2^2 + \| \nabla d^{R/B} \|_2^2 \quad \text{s.t. } d^X[k] = v[k], \ \forall k
\]

- Quadratic problem $\rightarrow$ linear system to solve
- Iterative method (Jacobi)  

[Condat, IEEE ICIP, 2009]
Periodic CFAs with arbitrary colors

[Bayer]

[Hirakawa, IEEE ICIP 2008]
Proposed 2x3 CFA with 6 colors

- 6 couleurs $[1,\frac{1}{2},0], [0,\frac{1}{2},1], [\frac{1}{2},0,1], [\frac{1}{2},1,0], [0,1,\frac{1}{2}], [1,0,\frac{1}{2}]$

- Color isotropy: two chrominance channels modulated in quadrature at the same frequency

- Designed to maximize $\gamma^C, \gamma^L$

$$\omega_0 = \frac{2\pi}{3}$$

$$cfa^L[k] = \gamma^L,$$
$$cfa^{R/B}[k] = \gamma^C (-1)^{k_1} \sqrt{2} \sin(\omega_0 k_2 - \varphi)$$
$$cfa^{V/M}[k] = \gamma^C (-1)^{k_1} \sqrt{2} \cos(\omega_0 k_2 - \varphi)$$
Results

Bayer

Condat
Results

\[ \sigma = 40 \]
Conclusion

- Demosaicking by frequency selection
  - linear (simple, stable), fast, efficient
  - noisy case: just demosaick and denoise the luminance
  - variational interpretation: generic, extension to non-quadratic penalty

- Can be inserted in a realistic reconstruction pipeline using variance stabilization

- There are better alternatives to the Bayer CFA (more robust to aliasing and noise)
  - R,G,B aperiodic CFAs
  - 2x3 periodic CFA