A GENERIC VARIATIONAL APPROACH FOR DEMOSAICKING FROM AN ARBITRARY COLOR FILTER ARRAY

*Laurent Condat**

GREYC
6 Bd du Maréchal Juin
14050 CAEN Cedex, France

ABSTRACT

We propose a method to demosaick images acquired with a completely arbitrary color filter array (CFA). We adopt a variational approach where the reconstructed image has maximal smoothness under the constraint of consistency with the measurements. This optimization problem boils down to a large, sparse system of linear equations to solve, for which we propose an iterative algorithm. Although the approach is linear, it yields visually pleasing demosaicked images and provides a robust framework for comparing the performances of CFAs.

***Index Terms*—** Demosaicking, color filter array, variational reconstruction

1. INTRODUCTION

Color images are acquired by digital cameras using a sensor on which a *color filter array* (CFA) is overlaid [1]. The most popular is the Bayer CFA [1], but the design of alternative CFAs currently knows a renewed interest [2, 3, 4]. There is an extensive literature dealing with *demosaicking*, the problem of reconstructing a color image from the sensor measurements—see e.g. [1, 5, 6] and references therein—but almost always, acquisition with the Bayer CFA is assumed. Although the design of new CFAs can be based on theoretical considerations, the need exists for a generic demosaicking method, to compare the practical performances of CFAs. To our knowledge, only the demosaicking approach of Lukac *et al.* can be used for comparisons, under the limitation that the CFAs consist of R,G, and B filters [7]; thus, new designs like in [2, 3] are excluded. In this work, we present a new demosaicking method which can be applied to arbitrary CFAs, without any constraint on the colors of their filters or their arrangement—periodic or random. For this, in Sect. 2, we regularize the demosaicking problem by seeking a maximally smooth solution, while consistent with the measurements. This optimization problem boils down to a linear system to solve. For this, we propose in Sect. 3 a simple iterative algorithm, which converges to a visually satisfying demosaicked image after a few number of iterations. We give in Sect. 4 experimental results with several CFAs.

First of all, we introduce our notations. $*$ denotes the convolution and boldface letters denote vectors, e.g. $\mathbf{k} = [k_1, k_2]^{\mathrm{T}} \in \mathbb{Z}^2$. A color image is denoted by $\mathbf{u} = (\mathbf{u}[\mathbf{k}])_{\mathbf{k}\in\mathbb{Z}^2}$, where $\mathbf{u}[\mathbf{k}] \in [0, 1]^3$ is the color of the pixel at location $\mathbf{k}$.

We define the color image $\mathbf{im}$ as the ground truth to be estimated by the demosaicking process, while $\mathbf{cfa}$ denotes the CFA used for the acquisition. Then, the mosaicked image v is such that $v[\mathbf{k}] = \mathbf{im}[\mathbf{k}]^{\mathrm{T}}\mathbf{cfa}[\mathbf{k}]$, for every $\mathbf{k} \in \mathbb{Z}^2$.

2. VARIATIONAL FORMULATION

First, the demosaicked image $\mathbf{dem}$ should be *consistent* with the available measurements; that is, we impose that $\mathbf{dem}[\mathbf{k}]^{\mathrm{T}}\mathbf{cfa}[\mathbf{k}] = v[\mathbf{k}],\ \forall\mathbf{k} \in \mathbb{Z}^2$. To regularize this ill-posed inverse problem, we use variational principles. So, we look for the image minimizing some quadratic functional penalizing the lack of smoothness, subject to consistency.

It is well known that in natural images, the R,G,B components are not independent [8, 1]. Thus, we consider the orthonormal basis corresponding to luminance, red-green and blue-yellow chrominances, defined in the R,G,B basis as $\mathbf{L} = \frac{1}{\sqrt{3}}[1, 1, 1]^{\mathrm{T}}, \mathbf{C}_1 = \frac{1}{\sqrt{2}}[-1, 1, 0]^{\mathrm{T}}, \mathbf{C}_2 = \frac{1}{\sqrt{6}}[-1, -1, 2]$, respectively. u^L, u^{C_1} and u^{C_2} are the components of a color image $\mathbf{u}$ in this basis. They can be considered statistically independent for natural images [8]. Therefore, we adopt a regularization functional which is diagonal is the luminance/chrominance basis. Thus, our optimization problem takes the form:

$$\mathbf{dem} = \underset{\mathbf{u}}{\mathrm{argmin}}\ \mu\mathcal{Q}(u^L) + \mathcal{Q}(u^{C_1}) + \mathcal{Q}(u^{C_2}), \quad (1)$$

$$\text{subject to}\ \ \mathbf{u}[\mathbf{k}]^{\mathrm{T}}\mathbf{cfa}[\mathbf{k}] = v[\mathbf{k}],\ \forall\mathbf{k} \in \mathbb{Z}^2, \quad (2)$$

where $\mathcal{Q}(u) = \|\nabla u\|^2 = \langle u, u * r\rangle$ is the simplest semi-norm using the discrete laplacian

$$r = \frac{1}{4}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}. \quad (3)$$

*This work was performed during the stay of the author in the Helmholtz Zentrum München, supported by the Marie Curie Excellence Team Grant MEXT-CT-2004-013477, Acronym MAMEBIA, funded by the European Commission. Contact: laurent.condat@greyc.ensicaen.fr.

The parameter μ in (1) plays a crucial role; it controls the balance between the smoothness of the luminance and of the chrominance in the reconstructed image. If μ is close to zero, then $\mathbf{dem}$ will be close to a monochrome image, since all the high frequency energy of v will be assigned to dem^L. On the contrary, for $\mu = 1$, the regularization functional is diagonal in the R,G,B basis and for a CFA with R,G,B filters, the process amounts to reconstruct the R,G,B channels independently by interpolation, a naive solution known to yield bad results. Consequently, μ should be chosen relatively small in order to get a hue which is varying smoothlier than the luminance. This way, the inter-correlations between the color channels in natural images are automatically taken into account. There is no mathematical rule for choosing μ and the best value for a given CFA should be chosen empirically by trial and error, to give the best performances on test images.

With the convex regularization functional in (1), the problem is well-posed: it is stable and has a unique solution. A more complex filter r can be used instead of (3), but we found empirically that this yields almost no quality improvement. We remark that the regularization is independant of the choice of the basis $\mathbf{C}_1, \mathbf{C}_2$; thus, the smoothness of the chrominance is fairly penalized, without privileged color axis.

2.1. Derivation of the Solution

When minimizing a quadratic criterion under a linear constraint, it is well known that the solution can be derived by expressing the associated Lagrangian criterion $\mathcal{C}(\mathbf{u})$ [9]:

$$\mathcal{C}(\mathbf{u}) = \mu\mathcal{Q}(u^L) + \mathcal{Q}(u^{C_1}) + \mathcal{Q}(u^{C_2}) \tag{4}$$

$$+ 2\sum_{\mathbf{k}\in\mathbb{Z}^2} \lambda[\mathbf{k}]\Big(\mathbf{u}[\mathbf{k}]^{\mathrm{T}}\mathbf{cfa}[\mathbf{k}] - v[\mathbf{k}]\Big). \tag{5}$$

Then, the desired solution $\mathbf{dem}$ is obtained by setting the partial derivatives of $\mathcal{C}$ to zero, with respect to the unknowns $u^L[\mathbf{k}]$, $u^{C_1}[\mathbf{k}]$, $u^{C_2}[\mathbf{k}]$ and the Lagrangian parameters $\lambda[\mathbf{k}]$. Thus, the demosaicked image is the solution to the following set of linear equations: for every $\mathbf{k} \in \mathbb{Z}^2$,

$$\begin{cases} \mu\,(\mathrm{dem}^L * r)[\mathbf{k}] + \lambda[\mathbf{k}]\mathrm{cfa}^L[\mathbf{k}] = 0, \\ (\mathrm{dem}^{C_1} * r)[\mathbf{k}] + \lambda[\mathbf{k}]\mathrm{cfa}^{C_1}[\mathbf{k}] = 0, \\ (\mathrm{dem}^{C_2} * r)[\mathbf{k}] + \lambda[\mathbf{k}]\mathrm{cfa}^{C_2}[\mathbf{k}] = 0, \\ \mathrm{dem}^L[\mathbf{k}]\mathrm{cfa}^L[\mathbf{k}] + \mathrm{dem}^{C_1}[\mathbf{k}]\mathrm{cfa}^{C_1}[\mathbf{k}] \\ \qquad + \mathrm{dem}^{C_2}[\mathbf{k}]\mathrm{cfa}^{C_2}[\mathbf{k}] = v[\mathbf{k}]. \end{cases} \tag{6}$$

In the next section, we propose an iterative algorithm to solve this system of linear equations.

3. PRACTICAL DEMOSAICKING METHOD

Since the linear system (6) does not have a simple sparse form, which would provide us with a direct solution, we propose an iterative scheme that converges to the solution. Formally, the method is a so-called Jacobi iterator, which approximates the inverse of the convolution matrix corresponding to r by the inverse of its diagonal [10]. That is, each refinement step updates the demosaicked image as follows:

$$\mathrm{dem}^L_{(n)}[\mathbf{k}] = \mathrm{dem}^L_{(n-1)}[\mathbf{k}] - \Big((\mathrm{dem}^L_{(n-1)} * r)[\mathbf{k}] + \lambda_{(n)}[\mathbf{k}]\mathrm{cfa}^L[\mathbf{k}]/\mu\Big), \tag{7}$$

$$\mathrm{dem}^{C_1}_{(n)}[\mathbf{k}] = \mathrm{dem}^{C_1}_{(n-1)}[\mathbf{k}] - \Big((\mathrm{dem}^{C_1}_{(n-1)} * r)[\mathbf{k}] + \lambda_{(n)}[\mathbf{k}]\mathrm{cfa}^{C_1}[\mathbf{k}]\Big), \tag{8}$$

$$\mathrm{dem}^{C_2}_{(n)}[\mathbf{k}] = \mathrm{dem}^{C_2}_{(n-1)}[\mathbf{k}] - \Big((\mathrm{dem}^{C_2}_{(n-1)} * r)[\mathbf{k}] + \lambda_{(n)}[\mathbf{k}]\mathrm{cfa}^{C_2}[\mathbf{k}]\Big), \tag{9}$$

for every $\mathbf{k} \in \mathbb{Z}^2$, where $X_{(n)}$ denotes the value of the variable X after the n-th iteration. With this scheme, the unknowns are spatially uncoupled. Then, our iterative algorithm boils down to solving, during each iteration and for each $\mathbf{k}$ in scanline order in the image, a 4×4 linear system in terms of the unknowns $\mathrm{dem}^L_{(n)}[\mathbf{k}]$, $\mathrm{dem}^{C_1}_{(n)}[\mathbf{k}]$, $\mathrm{dem}^{C_2}_{(n)}[\mathbf{k}]$, $\lambda_{(n)}[\mathbf{k}]$:

$$\begin{cases} \mathrm{dem}^L_{(n)}[\mathbf{k}] + \lambda_{(n)}[\mathbf{k}]\mathrm{cfa}^L[\mathbf{k}]/\mu = (\mathrm{dem}^L_{(n-1)} * r')[\mathbf{k}], \\ \mathrm{dem}^{C_1}_{(n)}[\mathbf{k}] + \lambda_{(n)}[\mathbf{k}]\mathrm{cfa}^{C_1}[\mathbf{k}] = (\mathrm{dem}^{C_1}_{(n-1)} * r')[\mathbf{k}], \\ \mathrm{dem}^{C_2}_{(n)}[\mathbf{k}] + \lambda_{(n)}[\mathbf{k}]\mathrm{cfa}^{C_2}[\mathbf{k}] = (\mathrm{dem}^{C_2}_{(n-1)} * r')[\mathbf{k}], \\ \mathrm{dem}^L_{(n)}[\mathbf{k}]\mathrm{cfa}^L[\mathbf{k}] + \mathrm{dem}^{C_1}_{(n)}[\mathbf{k}]\mathrm{cfa}^{C_1}[\mathbf{k}] + \\ \qquad \mathrm{dem}^{C_2}_{(n)}[\mathbf{k}]\mathrm{cfa}^{C_2}[\mathbf{k}] = v[\mathbf{k}]. \end{cases} \tag{10}$$

where $r' = \delta_{\mathbf{0}} - r$ and $\delta_{\mathbf{0}}$ is the identity filter. For given n and $\mathbf{k}$, this linear system is solved by first calculating $\lambda_{(n)}[\mathbf{k}]$ and then updating the pixel values of $\mathbf{dem}$ using the first three equations of the system (10). $\lambda_{(n)}[\mathbf{k}]$ is computed using the following equality, derived by substitutions in the system:

$$\lambda_{(n)}[\mathbf{k}]\Big(\mathrm{cfa}^L[\mathbf{k}]^2/\mu + \mathrm{cfa}^{C_1}[\mathbf{k}]^2 + \mathrm{cfa}^{C_2}[\mathbf{k}]^2\Big) = \mathrm{cfa}^L[\mathbf{k}](\mathrm{dem}^L_{(n-1)} * r')[\mathbf{k}] + \mathrm{cfa}^{C_1}[\mathbf{k}](\mathrm{dem}^{C_1}_{(n-1)} * r')[\mathbf{k}] + \mathrm{cfa}^{C_2}[\mathbf{k}](\mathrm{dem}^{C_2}_{(n-1)} * r')[\mathbf{k}] - v[\mathbf{k}]. \tag{11}$$

The proposed formalism is versatile, in the sense that it can handle *dead pixels*; that is, when the value $v[\mathbf{k}]$ is irrelevant for some $\mathbf{k}$. In that case, just set $\lambda_{(n)}[\mathbf{k}] = 0$ during the computation of $\mathbf{dem}_{(n)}[\mathbf{k}]$. This way, this value is computed so as to maximize the smoothness in the neighborhood of $\mathbf{k}$.

We also remark that the consistency is satisfied exactly by the image $\mathbf{dem}_{(n)}$ after each iteration. Thus, even only one iteration could be used as post-processing to improve the result of another demosaicking algorithm, which would yield visually pleasing but not consistent demosaicked images.

Since the solution of the problem is unique, the initial estimate $\mathbf{dem}_{(0)}$ only influences the speed of convergence. A choice that turns out to work well in practice consists in starting with a uniform grey image $\tilde{\mathbf{u}}_{(0)}[\mathbf{k}] = 127.5$ and in performing a few (we used 10) iterations with the parameter $\mu = 1$. Thus, the spatial distribution of the color information is roughly but quickly recovered and the repartition of the high frequency content between the luminance and chrominance channels is then refined by the subsequent iterations with the correct value of μ.

The algorithm is simple to implement and quite fast, since the complexity is about three convolutions with r at each iteration. However, we use a very simple iterative strategy, so that the convergence speed is relatively slow; that is, several hundreds of iterations are necessary to converge to the solution up to machine precision. But we found experimentally that the MSE with respect to the ground-truth $\mathbf{im}$ is generally minimized after a much smaller number of iterations, which depends on the CFA. Thus, for the tests in Sect. 4, we ran only 20 iterations for the CFAs (I)–(II) and 100 iterations for the CFAs (III)–(VI), including the 10 first iterations with $\mu = 1$.

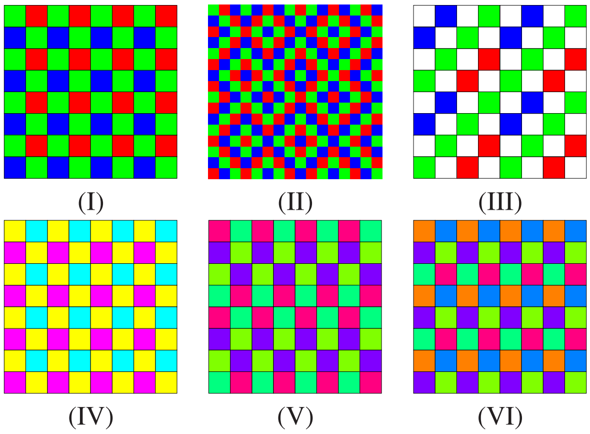

Fig. 1. The six CFAs used in the experiments of Sect. 4.

4. EXPERIMENTAL VALIDATION

For our tests, we considered the data set of 20 color images of size 768×512 used by many authors to test their methods (e.g. [11, 2]). These images were mosaicked using several CFAs and demosaicked using our method. The CFAs considered, depicted in Fig. 1 are: (I) the Bayer CFA, (II) the random pattern proposed in [4], (III) the new Kodak CFA [12], (IV) the CMY CFA, (V) the CFA of Hirakawa *et al.* [2] and (VI) our recently designed CFA [3]. By lack of space, we only report in Tab. 1 the mean squared error (MSE)[1] averaged over the 20 images. We used the values $\mu = 0.04, 0.04, 0.07, 0.12, 0.12, 0.10$ for the CFAs (I) to (VI), respectively. These values were roughly determined empirically to minimize the average MSE.

We compared our method to two other demosaicking techniques, usable only for CFAs with R,G,B filters: 1) The simple linear scheme, which consists in computing a missing value for the color $\mathbf{C} \in \{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$ at location $\mathbf{k}$, by averaging the pixel values $v[\mathbf{l}]$ for $\mathbf{l}$ in a 3×3 neighborhood surrounding $\mathbf{k}$ such that $h[\mathbf{l}] = \mathbf{C}$. This reverts to bilinear interpolation for the Bayer pattern. 2) The non-linear universal demosaicking algorithm of Lukac *et al.* [13] which is, to our knowledge, the only advanced demosaicking algorithm which can be used for every R,G,B CFA. The MSE results are summarized in Tab. 1. We observe that the simple bilinear method, which does not exploit the cross-correlations between the color bands in natural images, provides poor results. We found out that our linear approach generally outperforms the non-linear method of Lukac *et al.* [13], except for the Bayer CFA. This may indicate that their method has been designed to perform well with the Bayer CFA in particular. We also report in Tab. 1 the MSE obtained with demosaicking methods specifically tuned for the CFAs (I), (V),(VI), to show the lower bound achievable when fully exploiting the characteristics proper to a given CFA. When comparing the results of these methods with the ones of bilinear interpolation, we see that our method performs well. It does not compete with the state-of-the-art methods (see in Tab. 1 the result obtained for the Bayer CFA with the non-linear method in [6]), but this is the price to pay for having a generic method. We note that, to our knowledge, no advanced method has been proposed to date for the CFAs (III) and (IV), so that our approach is the only one available to evaluate their performances.

Our generic approach provides a convenient way to fairly compare the performances of CFAs. Since the goal of demosaicking is more to obtain visually pleasant images without artifacts that to estimate the ground-truth image optimally in the least-squares sense, the MSE results have to be balanced by a visual inspection of the demosaicked images. It is well-known that the Bayer CFA and its CMY counterpart exhibit a particularly high sensitivity to aliasing on horizontal or vertical patterns with high frequency content, like the fence in Fig. 2. The CFA (III) has the same problem, even more problematic because it occurs on patterns oscillating with a frequency two times lower than the Nyquist frequency. This is due to the 2×2 sparser distribution of the R,G,B filters than with the Bayer CFA. With the random CFA (II), the aliasing artifacts take the form of a colored high frequency noise randomly spread over the fence. Due to its low magnitude and the low sensitivity of the human visual system to such patterns, these artifacts are far less disturbing than the coherent fringes that appear with periodic CFAs. So, the superiority of random patterns with blue-noise properties over periodic arrangements with R,G,B filters is confirmed. More generally, this works shows that there is a significant margin of possible improvement over the Bayer CFA. The new designs of [2, 3] are very attractive; they are more robust to noise, due to their better light sensitivity, and significantly less sensitive to aliasing. Like in the example of Fig. 2, artifacts are virtually never visible with these CFAs. The even higher light sensitivity of the CFAs (III) and (IV) is obtained at the expense of a significantly degraded image quality.

[1]We do not take into account the first and last five rows and columns of the demosaicked images for the computation of the MSE.

CFA	Bayer (I)				random (II)			(III)	(IV)	(V)		(VI)	
method	bilin.	[13]	[6]	∗	bilin.	[13]	∗	∗	∗	[2]	∗	[3]	∗
av. MSE	91.37	11.88	8.54	12.49	89.16	17.86	12.13	18.76	13.26	8.07	11.30	7.50	10.01

Table 1. Average MSE over the test set of 20 images for several combinations of CFAs and demosaicking methods. ∗ denotes the proposed demosaicking method.

CFA (I) CFA (II) CFA (III) CFA (VI)

Fig. 2. Results of our demosaicking method on a part of the *Lighthouse* image, for some of the CFAs considered.

5. CONCLUSION

We proposed a new framework for image demosaicking, which is based on the minimization of a variational functional under the constraint of consistency with the available raw data. This formalism is linear, hence robust, and generic, since the CFA can be completely arbitrary. We proposed a simple and stable algorithm to implement our method. Although it is iterative, good results are obtained after a few iterations only. The method can be extended to the more realistic situation where the data are corrupted by additive noise, by relaxing the consistency using Lagrangian regularization. It can also be extended to demosaick multi-spectral images having more than three bands, e.g. for remote sensing applications. Moreover, it is worth mentioning that our formalism is an extension of the reconstruction framework in [14] to color images. Future works will investigate the possible extensions to non-quadratic smoothness penalties and the links with other regularization approaches of the literature like [15, 16].

6. REFERENCES

[1] B. K. Gunturk, J. Glotzbach, Y. Altunbasak, R. W. Schaffer, and R. M. Mersereau, "Demosaicking: Color filter array interpolation," *IEEE Signal Processing Mag.*, vol. 22, no. 1, pp. 44–54, Jan. 2005.

[2] K. Hirakawa and P. J. Wolfe, "Spatio-spectral color filter array design for optimal image recovery," *IEEE Trans. Image Processing*, vol. 17, no. 10, pp. 1876–1890, Oct. 2008.

[3] L. Condat, "A new color filter array with optimal sensing properties," in *Proc. of IEEE ICIP*, Nov. 2009.

[4] ——, "A new random color filter array with good spectral properties," in *Proc. of IEEE ICIP*, Nov. 2009.

[5] D. Alleyson, S. Süsstrunk, and J. Herault, "Linear demosaicing inspired by the human visual system," *IEEE Trans. Image Processing*, vol. 14, no. 4, pp. 439–449, Apr. 2005.

[6] E. Dubois, "Frequency-domain methods for demosaicking of Bayer-sampled color images," *IEEE Signal Processing Lett.*, vol. 12, no. 12, pp. 847–850, Dec. 2005.

[7] R. Lukac and K. N. Plataniotis, "Color filter arrays: Design and performance analysis," *IEEE Trans. Consumer Electron.*, vol. 51, no. 4, pp. 1260–1267, Nov. 2005.

[8] Y. Hel-Or, "The canonical correlations of color images and their use for demosaicing," HP Laboratories Israel, Tech. Rep. HPL-2003-164R1, Feb. 2004.

[9] G. Arfken, "Lagrange multipliers," in *Mathematical Methods for Physicists*, 3rd ed. Orlando, FL: Academic Press, 1985, pp. 945–950.

[10] W. Hakbusch, *Iterative Solutions of Large Sparse Systems of Equations*. New York: Springer-Verlag, 1994.

[11] B. K. Gunturk, Y. Altunbasak, and R. M. Mersereau, "Color plane interpolation using alternating projections," *Proc. IEEE*, vol. 11, no. 9, pp. 997–1013, Sept. 2002.

[12] T. Kijima, H. Nakamura, J. T. Compton, J. F. Hamilton, and T. E. DeWeese, "Image sensor with improved light sensitivity," U.S. Patent 0 268 533, Nov., 2007.

[13] R. Lukac and K. N. Plataniotis, "Universal demosaicking for imaging pipelines with a RGB color filter array," *Pattern Recognition*, vol. 38, pp. 2208–2212, 2005.

[14] M. Arigovindan, M. Sühling, P. Hunziker, and M. Unser, "Variational image reconstruction from arbitrarily spaced samples: A fast multiresolution spline solution," *IEEE Trans. Image Processing*, vol. 14, no. 4, pp. 450–460, Apr. 2005.

[15] T. Saito and T. Komatsu, "Demosaicing approach based on extended color total-variation regularization," in *Proc. of IEEE ICIP*, Oct. 2008.

[16] D. Menon and G. Calvagno, "Regularization approaches to demosaicking," *IEEE Trans. Image Processing*, in press, 2009.