

A SIMPLE, FAST AND EFFICIENT APPROACH TO DENOISAICKING: JOINT DEMOSAICKING AND DENOISING

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ABSTRACT

In most digital cameras, color images are captured by a sensor overlaid by the Bayer color filter array (CFA). *Denoisaicking* (joint demosaicking and denoising) consists in reconstructing a color image from the noisy “Bayerized” data output by the sensor. We show that the frequency analysis of the sampling pattern induced by the Bayer CFA provides a simple way to reconstruct the luminance and chrominance channels of the image. The process is reduced to adequate linear filtering operations and denoising of the grayscale luminance image.

Index Terms— Demosaicking, denoising, Bayer color filter array, frequency selection, spatio-spectral sampling

1. INTRODUCTION

Color images are usually acquired by digital cameras using a single sensor on which a color filter array (CFA) is overlaid [1]. The most popular is the Bayer CFA, which equips almost all cameras nowadays. In order to reconstruct a full-color image from the raw data delivered by the sensor, an interpolation process called demosaicking is performed. There is an abundant literature on demosaicking and we direct the readers to the broad and recent survey by Menon [2]. However, most demosaicking methods are developed under the unrealistic assumption of noise-free data. In the presence of noise, the performances of the algorithms degrade drastically, since their sophisticated nonlinear mechanisms are generally not robust to noise. Moreover, denoising after demosaicking is untractable, because demosaicking distorts the characteristics of the noise in a complex and hardly computable form. Thus, the joint problem of demosaicking and denoising the raw images of cameras, for which we coin the term *denoisaicking*¹, has to be handled as a whole [3, 4, 5, 6, 7, 8].

The denoisaicking problem can be formulated as follows. Let us first introduce some notations. Boldface letters denote vectors, e.g. $\mathbf{k} = [k_1, k_2]^T \in \mathbb{Z}^2$. We define the color image $\mathbf{u} = (\mathbf{u}[\mathbf{k}])_{\mathbf{k} \in \mathbb{Z}^2}$ as the ground-truth to be estimated by denoisaicking. For every \mathbf{k} , $\mathbf{u}[\mathbf{k}] = [u^R[\mathbf{k}], u^G[\mathbf{k}], u^B[\mathbf{k}]]^T$ is the color of the pixel of \mathbf{u} at location \mathbf{k} . In this paper, we adopt an

additive white Gaussian noise (AWGN) model; that is, we have at our disposal the *noisaicked* image v such that

$$v[\mathbf{k}] = u^X[\mathbf{k}] + \varepsilon[\mathbf{k}], \quad \forall \mathbf{k} \in \mathbb{Z}^2, \quad (1)$$

where $X \in \{R, G, B\}$ is the color of the filter in the Bayer pattern at location \mathbf{k} (see Fig. 1a), $\varepsilon[\mathbf{k}] \sim \mathcal{N}(0, \sigma^2)$ for every \mathbf{k} and σ^2 is the noise variance. In real conditions, the AWGN assumption is not met; the real noise is more accurately modeled by the sum of a Gaussian component and a signal-dependent Poissonian component and by taking into account the clipping due to the limited dynamic range of the sensor [9]. Moreover, the observed values are the photon counts, which have to be tone mapped/gamma corrected and this step modifies the noise characteristics. However, homomorphic nonlinear transformations can be efficiently employed for variance stabilization [10], so that the problem can be recast in the AWGN context.

The article is organized as follows. The sequel of this section is devoted to a brief survey of works on denoisaicking. In sect. 2, we recall the spatio-spectral model of Bayer sampling of Alleysson *et al.* and the demosaicking approach by frequency selection of Dubois. We extend these principles to the noisy case and describe the proposed denoisaicking method in sect. 3. It is validated by experiments in sect. 4.

Hirakawa and Parks addressed denoisaicking with a total least squares approach [3]. A different method consists in applying a wavelet transform to the noisy data. Then, the wavelet coefficients of the luminance and chrominance components are estimated and denoised [4]. In [5], the color differences are estimated with a MMSE approach that exploits both spectral and spatial correlations, to simultaneously decrease the noise and the interpolation error. Then, the CFA channel-dependent noise is removed with a wavelet-based approach. In [6], Paliy *et al.* proposed to decorrelate estimates of the color intensities by a color transformation, then to directionally interpolate and denoise them using anisotropic adaptive filters. In [7], directional varying-scale joint denoising/interpolation filtering kernels are applied directly on the noisaicked image. This approach performs worse than the one in [6], by the same authors, although it is faster. More recently, Menon *et al.* designed space-varying filters to minimize a quadratic regularization term [8]. The demosaicked image is denoised by thresholding the coefficients of its undecimated wavelet transform.

¹This neologism was used by K. Hirakawa and P. J. Wolfe during their tutorial talk at the conference ICIP’08.

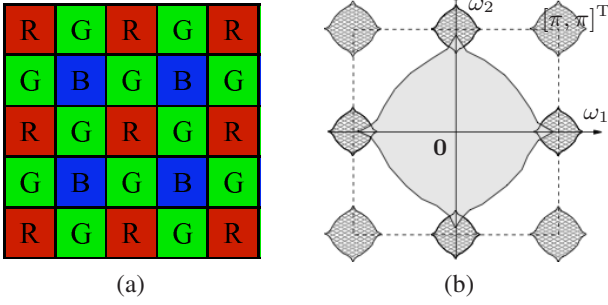


Fig. 1. (a) The Bayer CFA and (b) schematic representation of the spectrum of a mosaicked image, with the spectrum of the luminance u^L in the baseband and the replicas of the spectra of chrominance with hatched fill.

An alternative is to first denoise the noisaicked image before applying a classical demosaicking method. Traditional denoising approaches for grayscale images cannot be applied, because the mosaic structure violates the assumptions about local smoothness these methods rely on. Zhang *et al.* proposed a denoising method based on the principal component analysis (PCA) for shrinkage on image blocks [11]. In [12], the state-of-the-art BM3D denoising method [13] was extended to noisaicked images, using cross-color filtering and the special non-local modeling embedded in BM3D.

2. SPATIO-SPECTRAL MODEL OF NOISAICKING AND DEMOSAICKING BY FREQUENCY SELECTION

It is well known that the R,G,B components of natural images are strongly correlated [1]. That is why we define the components of luminance, green/magenta and red/blue chrominances of a color image \mathbf{a} as $a^L = \langle \mathbf{a}, \mathbf{L} \rangle$, $a^{G/M} = \langle \mathbf{a}, \mathbf{C}^{G/M} \rangle$, and $a^{R/B} = \langle \mathbf{a}, \mathbf{C}^{R/B} \rangle$, respectively, using the vectors $\mathbf{L} = \frac{1}{4}[1, 2, 1]^T$, $\mathbf{C}^{G/M} = \frac{1}{\sqrt{6}}[-1, 2, -1]^T$, $\mathbf{C}^{R/B} = \frac{1}{\sqrt{2}}[1, 0, -1]^T$. A major contribution of Alleysson *et al.* [14] consisted in showing that the basis $\mathbf{L}, \mathbf{C}^{G/M}, \mathbf{C}^{R/B}$ is appropriate to characterize the Bayer CFA and that the mosaicked image is the sum of the modulated luminance and chrominance components of \mathbf{u} . That is,

$$\hat{v}(\boldsymbol{\omega}) = \hat{u}^L(\boldsymbol{\omega}) + \frac{\sqrt{6}}{4}\hat{u}^{G/M}(\boldsymbol{\omega} - [\pi, \pi]^T) + \frac{\sqrt{2}}{4}\hat{u}^{R/B}(\boldsymbol{\omega} - [0, \pi]^T) - \frac{\sqrt{2}}{4}\hat{u}^{R/B}(\boldsymbol{\omega} - [\pi, 0]^T) + \hat{\varepsilon}(\boldsymbol{\omega}), \quad (2)$$

where the Fourier transform $\hat{a}(\boldsymbol{\omega})$ of an image a is defined as $\hat{a}(\boldsymbol{\omega}) = \sum_{\mathbf{k} \in \mathbb{Z}^2} a[\mathbf{k}]e^{-j\boldsymbol{\omega}^T \mathbf{k}}$ and an image with finite support is implicitly extended to an infinite one by zero-padding.

This frequency analysis of the spatio-spectral sampling induced by the Bayer CFA, illustrated in Fig. 1b, sheds an interesting light on denoising: it aims at separating the three images u^L , $u^{R/B}$ and $u^{G/M}$ from their noisy mixing in v . This is exactly what demosaicking by frequency selection does, in the noise-free case. This approach, proposed in its most rigorous form by Dubois [15, 16], consists in assigning the high fre-

quency content of v to the chrominance of the demosaicked image and the rest of the frequency content to the luminance. More precisely, the method consists in the following steps, where we denote convolutions by $*$ and the demosaicked image by \mathbf{d} .

1. Estimate the G/M chrominance by modulation and convolution with a lowpass filter $h_{G/M}$: $d^{G/M} = \frac{4}{\sqrt{6}}v_{\pi, \pi} * h_{G/M}$, where $v_{\pi, \pi}[\mathbf{k}] = (-1)^{k_1+k_2}v[\mathbf{k}]$.
2. Estimate the R/B chrominance by modulation and convolution with a lowpass filter $h_{R/B}$. Since this chrominance information is present in two exemplaries in \hat{v} , we obtain two estimates: $d_H^{R/B} = \frac{-4}{\sqrt{2}}v_{0, \pi} * h_{R/B}$ and $d_V^{R/B} = \frac{4}{\sqrt{2}}v_{0, \pi} * (h_{R/B})^T$, where $v_{\pi, 0}[\mathbf{k}] = (-1)^{k_1}v[\mathbf{k}]$, $v_{0, \pi}[\mathbf{k}] = (-1)^{k_2}v[\mathbf{k}]$, and $(h_{R/B})^T$ is the filter $h_{R/B}$ rotated by 90° . The two estimates $\text{dem}_H^{R/B}$ and $\text{dem}_V^{R/B}$ should then be fused. We consider the simplest fusion process, the linear average, which yields $d^{R/B} = \frac{1}{2}(d_H^{R/B} + d_V^{R/B})$. A nonlinear fusion was proposed in [15, 16], but we leave the study of its robustness to noise for future work.
3. Estimate the luminance as the residual information content of v by subtracting the re-modulated chrominance: $d^L = v - d_m^{G/M} - d_m^{R/B}$, where $d_m^{G/M}[\mathbf{k}] = \frac{\sqrt{6}}{4}(-1)^{k_1+k_2}d^{G/M}[\mathbf{k}]$ and $d_m^{R/B}[\mathbf{k}] = \frac{\sqrt{2}}{4}((-1)^{k_2} - (-1)^{k_1})d^{R/B}[\mathbf{k}]$.

The step 2. can be performed using a single spatially-varying convolution, so that this linear demosaicking method only reverts to two convolutions. Moreover, they can be performed in parallel. The filters $h_{R/B}$ and $h_{G/M}$ can be chosen using theoretical constraints [15]. Another approach [16] consists in calculating the filters minimizing the averaged mean squared error between \mathbf{d} and \mathbf{u} , for the images of a learning database.

3. PROPOSED DENOISAICKING APPROACH

Let us first analyze the behavior of demosaicking by frequency selection, as described in the previous section, in the presence of noise. We denote by v_0 the noise-free mosaicked image, so that $v[\mathbf{k}] = v_0[\mathbf{k}] + \varepsilon[\mathbf{k}]$ for every \mathbf{k} . We denote by \mathbf{d}_0 and \mathbf{d} the images produced by demosaicking, from v_0 and v , respectively. Due to the linearity of modulations and convolutions, we get $\mathbf{d} = \mathbf{d}_0 + \mathbf{e}$, where the components $e^{G/M}$, $e^{R/B}$, e^L of the color noise image \mathbf{e} are *independent*² Gaussian noise realizations. $e^{G/M}$ and $e^{R/B}$ are stationary and have spectral density $\frac{8}{3}\sigma^2|\hat{h}_{G/M}(\boldsymbol{\omega})|^2$ and $2\sigma^2(|\hat{h}_{R/B}(\omega_1, \omega_2)|^2 + |\hat{h}_{R/B}(\omega_2, \omega_1)|^2)$, respectively. So, a point to keep in mind is

²Rigorously, the independence is only approximative. The three noise realizations are independent if the functions $\hat{h}_{G/M}(\boldsymbol{\omega} - [\pi, \pi]^T)$, $\hat{h}_{R/B}(\omega_1 - \pi, \omega_2)$ and $\hat{h}_{R/B}(\omega_2 - \pi, \omega_1)$ are indicator functions with compact and disjoint supports. In practice, since we use strongly lowpass filters $h_{G/M}$ and $h_{R/B}$, the independence is relatively well satisfied.

that the G/M and R/B chrominance basis is the right one to consider for image denoising, because in it, the two noise realizations are independent and have different variance. Considering another chrominance representation would make the noise correlated and difficult to remove subsequently.

However, the luminance noise e^L is not stationary and not white, because of the subtraction from v of the R/B chrominance re-modulated at the *two* frequencies $[\pi, 0]^T$ and $[0, \pi]^T$ in step 3. Therefore, it is very difficult to remove. We will see that the solution comes from denoising the chrominance channels *before* estimating the luminance.

For this, we keep the algorithm unchanged but seek the filters $h_{G/M}$ and $h_{R/B}$ that estimate the denoised chrominance directly. The method for calculating the least-squares filters for demosaicking, detailed in [16], can be modified to obtain the denoising filters which are optimal in the Wiener sense, for a given base of test images. Let us determine the filter $h_{G/M}$ of, say, size 9×9 . We denote by \mathbf{x} the vector containing the 81 coefficients of the filter, in lexicographic order; by \mathbf{A} the matrix whose each row contains the 81 pixel values of every 9×9 patch present in the set of learning images, after the images have been mosaicked and re-modulated (pointwise multiplication by $\frac{4}{\sqrt{6}}(-1)^{k_1+k_2}$); and by \mathbf{b} the vector containing the corresponding chrominance values, for the central pixel of each patch. We want to minimize the expectation of the error $\mathcal{E}\{\|\hat{\mathbf{A}}\mathbf{x} - \mathbf{b}\|^2\}$, where $\hat{\mathbf{A}}$ is equal to \mathbf{A} plus noise, since every re-modulated mosaicked patch is contaminated by white noise of variance $\frac{8}{3}\sigma^2$. We have: $\mathcal{E}\{\hat{\mathbf{A}}^T\mathbf{b}\} = \mathbf{A}^T\mathbf{b}$ and $\mathcal{E}\{\hat{\mathbf{A}}^T\hat{\mathbf{A}}\} = \mathbf{A}^T\mathbf{A} + \frac{8}{3}\sigma^2\mathbf{N}\mathbf{I}$, where \mathbf{I} is the identity matrix and N is the total number of pixels in the image base (the number of rows of \mathbf{A}). Therefore, the coefficients of the Wiener filter $h_{G/M}$ are given by the vector \mathbf{x} solution of the linear system of size 81×81 :

$$\left(\frac{1}{N}\mathbf{A}^T\mathbf{A} + \frac{8}{3}\sigma^2\mathbf{I}\right)\mathbf{x} = \frac{1}{N}\mathbf{A}^T\mathbf{b}. \quad (3)$$

This system can be formed easily by reading the images in scanline, without having to store the matrices \mathbf{A} and \mathbf{b} . The filter $h_{R/B}$ is obtained the same way, by solving a system like in eqn. (3), where the noise amplification gain $8/3$ is replaced by 4. Note that if $\sigma = 0$, we recover the least-squares filter design of Dubois [16]. For the experiments in the next section, we used symmetric filters of size 13×13 , since 9×9 is not enough for high noise levels. In practice, since the required filters are strongly lowpass, it is more appropriate and faster to use separable recursive filters instead of the FIR filters proposed here.

Since the chrominance images $d^{G/M}$ and $d^{R/B}$ are denoised, once they are re-modulated and subtracted from v during the step 3. of the method, the residual image is $d^L \approx u^L + \varepsilon$, according to (2). Therefore, the proposed denoising method simply consists in adding a step 4. to the demosaicking method by frequency selection:

4. Consider d^L as a grayscale image corrupted by AWGN of variance σ^2 and apply a denoising method to it.

By lack of place, we omit the mathematical analysis showing that the assumption of AWGN in d^L is a good approximation.

Denoising method	$\sigma = 1$	$\sigma = 10$	$\sigma = 20$
[3]	32.95	30.06	27.62
[5]	39.40	32.17	28.66
[6]	37.37	32.06	28.77
[11]+[17]	38.09	32.17	28.82
proposed	38.49	32.56	29.57
proposed+mosaicking+ [17]	39.42	32.73	29.67

Table 1. Average CPSNR (in dB) over the 24 images of the Kodak test set for different denoising methods.

4. EXPERIMENTAL VALIDATION

We compared our approach against methods of the literature for which code is provided by the authors on their homepage; that is, the methods of [3], [5], [6] and the denoising-first method of [11] combined with the nonlinear demosaicking method of [17]. The 24 color images of the classical Kodak test set³ were mosaicked⁴ with the Bayer CFA and corrupted with different noise levels⁵. The state-of-the-art BM3D denoising method [13] was used in step 4. of our approach, to illustrate its full potential. In Tab. 1, we report the CPSNR⁶ between \mathbf{u} and \mathbf{d} , averaged over the 24 images. We also propose an extended variant of our approach in which the denoised image is mosaicked again and then demosaicked using the method of [17]. Our approach outperforms the other methods compared. The extended variant provides a significant gain for low noise levels; this shows the superiority of good nonlinear demosaicking methods over linear frequency selection for noise-free mosaicked images. Visually, as can be seen in the example of Fig. 2, our method provides images with a natural look, without the structured artifacts typical of the other methods, like washed out textures, ringing artifacts near edges, wavy or brushed patterned noise or checkerboard patterns in homogeneous areas.

5. CONCLUSION

We proposed a simple denoising approach, which consists in recovering the chrominance by linear filtering and denoising the luminance using a classical method for grayscale images. A Matlab implementation of our approach has been made available online⁷. There exist efficient denoising methods, including the Non-Local Means, for which fast implementations have been reported [18], [19]. Using such an algorithm within our approach

³The database is available at <http://r0k.us/graphics/kodak/>. The vertical images were first rotated by 90° counterclockwise to simulate an acquisition with a digital camera whose sensor is aligned horizontally.

⁴The filter for the top-left pixel was green and its right neighbor was red.

⁵We used the same noise realization for every method and every image, using the Matlab command `randn('state', 0)`;

⁶The CPSNR between the color images \mathbf{u}_1 and \mathbf{u}_2 is equal to $10 \log_{10}(255^2/\text{MSE})$ with $\text{MSE} = \sum_{X=R,G,B} \sum_{\mathbf{k}} (u_1^X[\mathbf{k}] - u_2^X[\mathbf{k}])^2 / (3N.M)$ for images of size $N \times M$. A 20 pixel-wide band around the border of the images was ignored when computing the CPSNR, since some of the tested algorithms do not perform well in said band. The pixel values were rounded to integers in 0...255 before computing the CPSNR.

⁷at <http://www.greyc.ensicaen.fr/~lcondat/>



Fig. 2. From top left to bottom right: part of the ground-truth image no. 3 of the Kodak base, parts of the denoised images with the methods [3], [5], [6], [11]+[17], proposed. The noise level was $\sigma = 20$.

opens the door to real-time high-quality denoising. The proposed framework can also be extended to other CFAs [20].

6. REFERENCES

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