A COMPACT IMAGE MAGNIFICATION METHOD WITH PRESERVATION OF PREFERENTIAL COMPONENTS

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ABSTRACT

Image magnification has been attracting a great deal of attention for long, and many approaches have been proposed to date. Nevertheless, bicubic interpolation is still the standard approach since it can be easily computed and does not require *a priori* knowledge nor a complicated model. In spite of such convenience, the images enlarged bicubically are blurry, in particular for large magnification factors. In this paper, we propose a new method, which is as compact as bicubic interpolation, while performing better than it. The key technique we used in this method is a sampling theorem that preserves preferential components in input images. We show that, by choosing the edge enhancement components as the preferential components, the proposed method performs much better than bicubic interpolation, with the same, or even less amount of computation.

Index Terms— Image enhancement, image reconstruction, interpolation

1. INTRODUCTION

Image magnification is an important basic task of producing a digital image with a higher resolution than its source image. This is often required in situations such as focusing on regions of interest, or displaying or printing an image at a higher resolution than the original data. As seen in the excellent survey of [1], many approaches have been proposed to date. They include methods using pixel classification [2, 3], spatial neural networks [4], and sparse derivative prior [5]. These approaches use plenty of appropriate reference images. However, it is not always that such a large image database is available. Another interesting approach is use of anisotropic diffusion [6], but needs lots of extra computations.

Spline based interpolation is also used widely for image magnification [7]. These methods perform very well from the viewpoints of both quality and computational cost.

In spite of these recent studies, bicubic interpolation [8] is still the standard approach to image magnification. Along with easy implementation and fast computation, the advantages of this method include the fact that neither *a priori* knowledge nor reference image data is necessary. However, the enlarged images generally appear blurry, especially for large magnification factors.

In this paper, by relaxing or replacing constraints in bicubic interpolation, we propose a new magnification method, which performs better than bicubic interpolation, but retains its compactness. To this end, we use a sampling theorem that reproduces all pixel values in the original low resolution image, and reconstructs pre-determined preferential components in input images perfectly. The preferential components dominates the performance of the proposed method. In this paper, we use the DCT basis as the reconstruction basis, and edge enhancement components as the preferential components. It is shown by simulations using standard images that the proposed method outperforms bicubic interpolation with the same, or even less amount of computation.

2. MATHEMATICAL PRELIMINARIES

Let $N_x \times N_y$ be the number of pixels in the initial image from which we obtain a pixel value of the magnified image. In bicubic interpolation, $N_x = N_y = 4$. Consider an area occupied by these $N_x N_y$ pixels, and set the origin at the top left corner. The x and y axes are set from the origin to the right and the bottom, respectively. Without loss of generality, the sampling interval is considered to be one. Then, the width and the height of the area become N_x and N_y, respectively. The center of each pixel is denoted by (x_{n_x}, y_{n_y}) $(n_x = 1, 2, \ldots, N_x, n_y = 1, 2, \ldots, N_y)$. Let H be a function space which consists of all square integrable functions on $[0, N_x] \times [0, N_y]$. The inner product in H is defined by $\langle f, g\rangle = \frac{1}{N_x N_y} \int_0^{N_x} \int_0^{N_y} f(x, y) g(x, y) dx dy$.

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3. NEW IMAGE MAGNIFICATION METHOD

3.1. Estimation model

Let $\tilde{f}(x,y)$ be a modeling function in H which approximates a target continuous image $f(x,y)$ in H. In bicubic interpolation, the modeling function is always a cubic polynomial regardless of the magnification factor, which is denoted by M_R. On the other hand, we use the following model which varies depending on the factor:

$$\tilde{f}(x,y) = \sum_{k_x=1}^{K_x} \sum_{k_y=1}^{K_y} c_{k_x,k_y} \varphi_{k_x,k_y}(x,y), \tag{1}$$

where $K_x = \lfloor M_R N_x \rfloor$ and $K_y = \lfloor M_R N_y \rfloor$ with $\lfloor r \rfloor$ denoting the greatest integer not exceeding r. This model is used so that $\tilde{f}$ can represent details of a target image. Let $\mathcal{R}$ be a subspace spanned by $\{\varphi_{k_x,k_y}\}_{k_x=1}^{K_x}{}_{k_y=1}^{K_y}$, which we assume an orthonormal basis in $\mathcal{R}$ for simplicity.

A complex model can cause an unstable solution. In order to avoid this problem, we use a type of regularization techniques. This will be discussed in Section 3.4.

3.2. Sampling process

With bicubic interpolation, a pixel value, denoted by d_{n_x,n_y}, is assumed to be $f(x_{n_x}, y_{n_y})$, which is a value of the continuous unknown target image $f(x,y)$ at a sample point (x_{n_x}, y_{n_y}) (ideal sampling). However, practical data acquisition devices, such as CCD or CMOS, have spatial extent, that is a non-ideal impulse response. Therefore, we consider a generalized sampling model [9], in which a pixel value is expressed as an inner product between the target image f and a sampling function ψ_{n_x,n_y}:

$$d_{n_x,n_y} = \langle f, \psi_{n_x,n_y} \rangle$$

Let A_s be the sampling operator defined by

$$A_s f = \begin{pmatrix} \langle f, \psi_{1,1} \rangle \\ \langle f, \psi_{2,1} \rangle \\ \vdots \\ \langle f, \psi_{N_x,N_y} \rangle \end{pmatrix}.$$

The set of H consisting in the functions that map to 0 through A_s is called the null space of A_s, and denoted by $\mathcal{N}$.

As usual, we assume the sampling operation to be shift-invariant: $\psi_{n_x,n_y}(x,y) = \psi(x - x_{n_x}, y - y_{n_y})$. In order for this to be theoretically sound in the finite area $[0, N_x] \times [0, N_y]$, $\psi(x,y)$ is defined to be a function that has periods N_x and N_y along the x and y axes, respectively.

3.3. Number of pixels

The number $N_x \times N_y$ of pixels used for magnification by bicubic interpolation is fixed to 4×4. This is because four parameters are necessary and sufficient to uniquely determine a cubic polynomial. Since we use the estimation model in Eq. (1) combined with a regularization technique, we are not restricted to 4×4 pixels. Hence, we set $N_x = N_y$ as 3, 4, 5, and 6 in simulations later, and evaluate which one is the best from the viewpoints of both accuracy and computational cost.

3.4. Criterion

With bicubic interpolation, a cubic polynomial is determined so that it reproduces the center two pixels among 4 pixels, and the slopes at the two pixels agree with the slopes of lines connecting the adjacent two pixels, respectively. This criterion is not flexible to the change of N_x and N_y. Therefore, in this paper, we consider a criterion which requests the following two conditions.

1. Sampling results of $\tilde{f}$ agree with those of f.
2. Predetermined components of f are perfectly reconstructed in $\tilde{f}$.

The first condition is expressed as

$$\langle \tilde{f}, \psi_{n_x,n_y} \rangle = \langle f, \psi_{n_x,n_y} \rangle \ \ (= d_{n_x,n_y}). \tag{2}$$

Bicubic interpolation does not request Eq. (2) for pixels at the both ends, while we do for all $N_x N_y$ pixels.

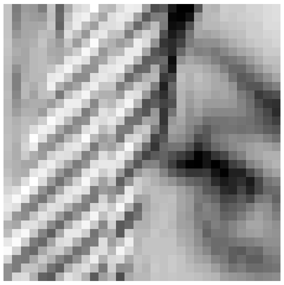
The number $K_x K_y$ of the terms of $\tilde{f}$ is mostly greater than $N_x N_y$ because $M_R > 1$. In this case, the first condition does not uniquely determine $\tilde{f}$. Then, we introduce a second condition in order to regularize the problem. Let $\{\phi_m^{(i)}\}_{m=1}^{M}$ ($M \leq N_x N_y$) be preferential components which are linear combinations of $\{\varphi_{k_x,k_y}\}_{k_x=1}^{K_x}{}_{k_y=1}^{K_y}$ in Eq. (1):

$$\phi_m^{(i)}(x,y) = \sum_{k_x=1}^{K_x} \sum_{k_y=1}^{K_y} c_{k_x,k_y}^{(m)} \varphi_{k_x,k_y}(x,y), \tag{3}$$

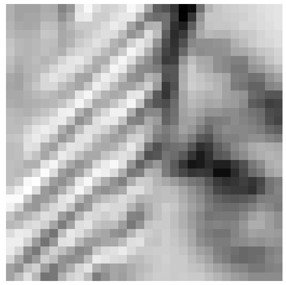
These components are members of $\mathcal{R}$, but must not belong to $\mathcal{N}$. Assume that $\{\phi_m^{(i)}\}_{m=1}^{M}$ are specified in advance.

If $M = N_x N_y$, then a linear combination of $\{\phi_m^{(i)}\}_{m=1}^{M}$ which satisfies Eq. (2) is unique. On the other hand, if $M < N_x N_y$, then there still remains a degree of freedom. Hence, we choose $N_x N_y - M$ functions $\{\phi_m^{(c)}\}_{m=M+1}^{N_x N_y}$, which are members of $\mathcal{R}$ but do not belong to $\mathcal{N}$, so we estimate $\tilde{f}$ by a linear combination of totally $N_x N_y$ functions of $\{\phi_m^{(i)}\}_{m=1}^{M}$ and $\{\phi_m^{(c)}\}_{m=M+1}^{N_x N_y}$. There can be various ideas about how to determine these functions. In this paper, we decide $\{\phi_m^{(c)}\}_{m=M+1}^{N_x N_y}$ so that these functions are orthogonal to both $\mathcal{N}$ and the subspace spanned by $\{\phi_m^{(i)}\}_{m=1}^{M}$.

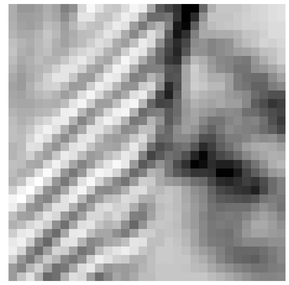
The problem of constructing $\tilde{f}(x,y)$ that satisfies the aforementioned conditions is a problem of sampling addressed by the first author in [10]. As discussed in this article, $\tilde{f}$ is the



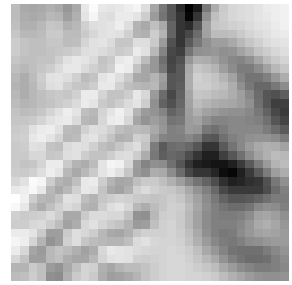
(a) Original image



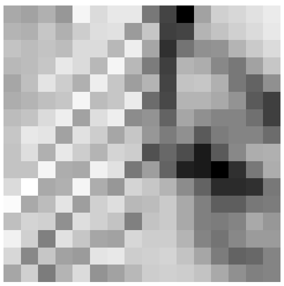
(c) Proposed ($M = 9$, $N_x = 3$)



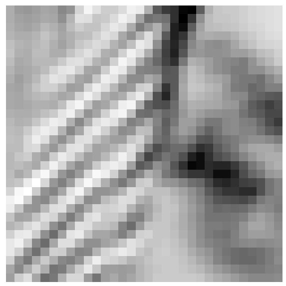
(e) Proposed ($M = 25$, $N_x = 5$)



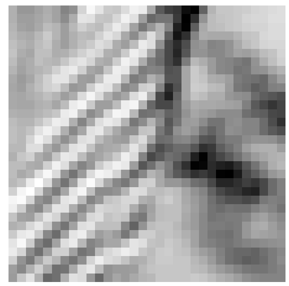
(g) Bicubic interpol.



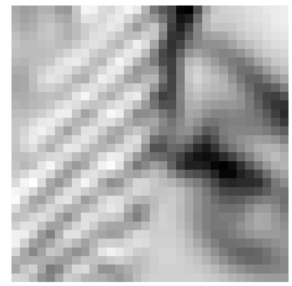
(b) Downsampled image



(d) Proposed ($M = 16$, $N_x = 4$)



(f) Proposed ($M = 36$, $N_x = 6$)



(h) Cubic spline interpol.

Fig. 1. Simulation results for the Barbara image with various methods (see text).

oblique projection of f along $\mathcal{N}$ onto a subspace spanned by $\{\phi_m^{(i)}\}_{m=1}^{M}$ and $\{\phi_m^{(c)}\}_{m=M+1}^{N_x N_y}$. This is the essential reason why the preferential components are perfectly reconstructed.

The oblique projection is constructed with the coefficients $\{c_{k_x,k_y}\}_{k_x=1}^{K_x}{}_{k_y=1}^{K_y}$ in Eq. (1), which are obtained as follows. Let $\boldsymbol{c}$ and $\boldsymbol{d}$ be vectors whose $k_x+(k_y-1)K_x$ and $n_x+(n_y-1)N_x$ elements are c_{k_x,k_y} and d_{n_x,n_y}, respectively. Let B and C be the cross-correlation matrix between $\{\varphi_{k_x,k_y}\}_{k_x=1}^{K_x}{}_{k_y=1}^{K_y}$ and $\{\psi_{n_x,n_y}\}_{n_x=1}^{N_x}{}_{n_y=1}^{N_y}$, and a $K_xK_y \times M$ matrix that consists of $c_{k_x,k_y}^{(m)}$ in Eq. (3), respectively. The adjoint matrix and the Moore-Penrose pseudo inverse of matrix T are denoted by T^* and $T^\dagger$, respectively.

Theorem 1 [10] Let U be a matrix defined by

$$U = B^* + CC^\dagger B^* - B^\dagger BC(C^* B^\dagger BC)^\dagger C^* B^*.$$

The function $\tilde{f}$ reconstructed by Eq. (1) with

$$\boldsymbol{c} = U(BU)^\dagger \boldsymbol{d}$$

satisfies Eq. (2), and perfectly reconstructs the predetermined components $\{\phi_m^{(i)}\}_{m=1}^{M}$ and $\{\phi_m^{(c)}\}_{m=M+1}^{N_xN_y}$.

Once $\tilde{f}$ is determined, the pixel value $h_{\gamma,\eta}$ in the high resolution image is obtained by the inner product $\langle \tilde{f}, \xi_{\gamma,\eta} \rangle$, where $\xi_{\gamma,\eta}(x, y)$ is the sampling function for high resolution image. This inner product is simply computed by

$$h_{\gamma,\eta} = \sum_{n_x=1}^{N_x} \sum_{n_y=1}^{N_y} \alpha_{n_x+(n_y-1)N_x}^{(\gamma,\eta)} d_{n_x,n_y}, \qquad (4)$$

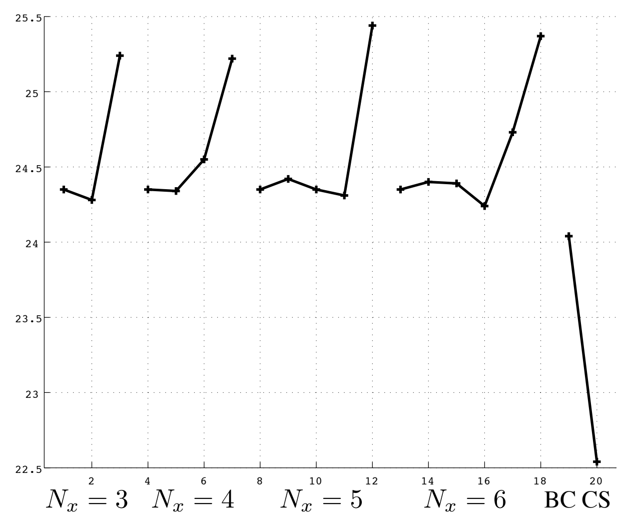


Fig. 2. PSNR values (dB) for the resizing experiments on the Barbara image.

where $\alpha_{n_x+(n_y-1)N_x}^{(\gamma,\eta)}$ is the element of the vector $\boldsymbol{\alpha}^{(\gamma,\eta)} = (U^*B^*)^\dagger U^* A_r \xi_{\gamma,\eta}$.

4. SIMULATIONS

We now present simulation results of the proposed method. The standard image shown in Fig. 1 (a) was used in the simulation. This image was downsized by a factor of two using 2×2 averaging. The resultant image is shown in (b). Magnification was performed on this image.

In the proposed method, we used the DCT basis as the reconstruction functions $\{\varphi_{k_x,k_y}\}_{k_x=1}^{K_x}{}_{k_y=1}^{K_y}$. That is, by using

$$\varphi_{k_x}(x) = \begin{cases} 1 & (k_x = 1), \\ \sqrt{2}\cos\dfrac{(k_x-1)\pi x}{l_x} & (k_x = 2, \ldots, K_x), \end{cases}$$

and the similarly defined $\varphi_{k_y}(y)$, the reconstruction function $\varphi_{k_x,k_y}(x,y)$ is given by $\varphi_{k_x}(x)\varphi_{k_y}(y)$. Since the DCT basis is orthonormal, we can use Theorem 1.

Sampling functions are assumed to be $\psi(x,y) = \psi(x)\psi(y)$, where

$$\psi(x) = \begin{cases} p_L & (|x| \leq r_L), \\ 0 & (|x| > r_L). \end{cases} \tag{5}$$

The sampling function for high resolution image is the same as Eq. (5) except that its support is half of r_L and the constant value in the support is twice of p_L. These parameters are set to be $r_L = 1/2$ and $p_L = 1/(2r_L)$.

Let $\tilde{\phi}_{k_x,k_y}$ be functions defined by $\tilde{\phi}_{1,1}(x,y) = 1$, and

$$\tilde{\phi}_{k_x,k_y}(x,y) = \sqrt{2}\cos\pi\left(\frac{k_x-1}{l_x}x + \frac{k_y-1}{l_y}y\right)$$

for $k_x > 1$ or $k_y > 1$. These functions represent fringe patterns with angles determined by k_x and k_y, and hence well approximate edge boundaries in images. Unfortunately, these functions do not belong to V_r. Hence, we used orthogonal projections of $\tilde{\phi}_{k_x,k_y}$ onto V_r as preferential components $\{\phi_m^{(i)}\}_{m=1}^{M}$. More precisely, since we can specify up to N_xN_y components, we used orthogonal projection of $\{\tilde{\phi}_{1,1}\}$ ($M = 1$), $\{\tilde{\phi}_{1,1}, \tilde{\phi}_{2,1}, \tilde{\phi}_{1,2}, \tilde{\phi}_{2,2}\}$ ($M = 4$), $\cdots$, and $\{\tilde{\phi}_{1,1}, \tilde{\phi}_{2,1}, \tilde{\phi}_{1,2}, \ldots, \tilde{\phi}_{N_x,N_x}\}$ ($M = N_x^2$). For comparison, we also magnified the downsized image by bicubic interpolation (BC) and cubic spline interpolation (CS).

The PSNR values in dB obtained by the proposed methods with $M = 1, 4, \ldots, N_x^2$ for each $N_x = 3, 4, 5, 6$ are shown in Fig. 2, with ones by bicubic interpolation and cubic spline interpolation. The horizontal axis shows the methods (the proposed methods with different M and N_x, BC, and CS), while the vertical axis shows the corresponding PSNR values. We can see that the PSNR value increases as M increases for each N_x. The magnified image by the proposed method with $M = N_x^2$ and $N_x = 3 \sim 6$ are shown in (c) $\sim$ (f), respectively. We can see that the texture pattern is clearly reconstructed in every image. Although the PSNR values are a little different ones from the others, the subjective visual quality is quite similar. The magnified images by bicubic interpolation and cubic spline interpolation are shown in (g) and (h), respectively. Bicubic interpolation does not reconstruct the texture pattern. Cubic spline interpolation reconstructed it better than bicubic interpolation, but somehow yielded a smaller PSNR value.

As shown in Eq. (4), the proposed method needs N_xN_y multiplications irrespective of M to produce one pixel in the magnified image. This means that the proposed method with $N_x = 4$ is done by the same amount of computation as bicubic interpolation. Further, that with $N_x = 3$ needs less amount of computation. Hence, we can conclude that the proposed methods achieve better image magnification with the same or even less amount of computation.

It is worthy to note that the edge enhancement functions as the preferential components work very effectively for texture regions with structured patterns like the stripes in our test image. However, the preferential components have generally to be chosen appropriately, depending on the local characteristics of the image (in our test, the eyebrow with orientation different from the stripes appears aliased).

5. CONCLUSION

We proposed an image magnification method based on a sampling theorem that preserves preferential components in input images. We showed that, by choosing edge enhancement functions as the preferential components, the proposed method performs much better than bicubic interpolation with less computational cost. We are currently focusing on the problem of determining the preferential components automatically depending on local characteristics of the image.

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